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Dynamic Interactions of Capital, Environment and Renewable Resources with Environment and Resource Policies

Wei-Bin Zhang
Ritsumeikan Asia Pacific University, Japan

Abstract: This paper proposes a dynamic economic model with endogenous physical capital, pollution, and renewable resources. The model is a synthesis of the neoclassical growth theory and the traditional dynamic models of resource and environmental economics with an alternative approach to household behaviour. The model describes a dynamic interdependence among physical accumulation, environmental dynamics, resource change, and division of labour in competitive markets under government intervention with environmental protection. Because of its refined economic structure, the model shows some interactions among economic variables which are not found in the existing literature of economics within a single analytical framework with a microeconomic foundation. We simulate the model to demonstrate existence of equilibrium points and motion of the dynamic system. Our comparative dynamic analysis shows, for instance, that a rise in the tax rate on the consumption of a good will reduce the consumption of the good in the short term and increase its consumption in the long term; efforts towards improving the environment are increased both in terms of capital and labour inputs; resource stock is reduced and its price is increased over time while resource consumption is increased over time; total capital and capital input of the environment sector are increased over time with capital inputs of sectors related to resource and good falling initially but rising soon; both the rate of interest and wage rate rise over time; the labour force of the environmental sector is increased over time but the labour force of the production sector is reduced over time; finally the labour force of the resource sector rises initially, then falls, and rises again.

Keywords: Capital accumulation, environmental change, harvesting, pollution, renewable resource, taxes on consumption and production
JEL classification: Q27, Q57

1. Introduction

The most salient feature of contemporary economics is the increasingly complicated interdependence among economic growth, economic structural changes, international trade, environmental change, and resource dynamics over time and space. Moreover, the role of the government in the complexity of these dynamic interactions is changing rapidly in different parts of the world. To understand these dynamic interactions, it is crucial to take account of public spending and different taxes. In recent years, environmental issues have received more attention than ever. Environmental issues have increasingly raised attention in the literature of economic growth and development. As mentioned by Tsurumi and Managi (2010), one can find three effects that are important in explaining the level of environmental pollution and resource use: (i) increases in output tends to require more inputs and produce

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Ritsumeikan Asia Pacific University, Jumonjibaru, Beppu-Shi, Oita-ken0874-8577, Japan.
Email: *wbz1@apu.ac.jp*
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more emissions; (ii) changes in income or preferences may lead to policy changes which will affect production and thus emission; and (iii) as income increases, the economic structure may be changed which will causes changes in the environment (see also Kijima *et al*., 2010). It is argued that the net effect of these effects tends to result in the environmental Kuznets curve. Nevertheless, a large number of empirical studies on the environmental Kuznets curve for various pollutants find different relations - for instance, inverted U-shaped relationship, a U-shaped relationship, a monotonically increasing or monotonically decreasing relationship between pollution and rising per capita income levels (Tsurumi and Managi 2010). The ambiguous or situation-dependent relations between environmental quality and economic growth and the inability of economic growth theory to properly explain these observed phenomena implies the necessity of building more comprehensive economic theories. Another dynamics, which is not properly analysed in economic growth theory is resource. Indeed, the scarcity of natural resources has been introduced into the neoclassical growth theory as early as in the 1970s (e.g., Stiglitz 1974), even though economists had been aware of the necessity of modeling resources with dynamic theory long before. Nevertheless, as pointed out by Munro and Scott (1985), before the 1960s, it was quite difficult to develop workable dynamic models of resources. After the 1970s, some economic growth models with dynamics of resources were proposed. Solow (1999) emphasised the importance of introducing natural resources into the neoclassical growth theory. Nevertheless, Solow did not address an important issue,that is, how to incorporate consumption of renewable resource into the growth model.

As pointed out by Fullerton and Kim (2008), existing research has proposed a number of different models for analysing different questions not in an integrated way. For instance, most growth models in mainstream economics are mainly concerned with physical capital or/and human capital accumulation, without explicitly introducing environment or/and resources as endogenous variables. The purpose of this study is to examine interactions among economic growth, environmental change and resource dynamics. This paper integrates various ideas in the literature of economic growth, environmental economics and resource economics into a single framework. This paper combines the economic mechanisms in the neoclassical growth theory, the logistic model of renewable resource in resource economics and the model of dynamic pollution in environmental economics into a single comprehensive framework. The model is a synthesis of Zhang's growth models with environmental change (Zhang 2011) and renewable resource dynamics (Zhang 2011a). The main contribution of this paper is to synthesise the economic mechanisms in Zhang's previous two models in a comprehensive framework. By doing this, this paper finds some interactions among environment, resource and economic growth, which were not revealed in Zhang's previous works. The paper is organised as follows. Section 2 introduces the basic model. The model describes a dynamic interdependence between wealth accumulation, environmental change, resource dynamics, and division of labour. Section 3 examines dynamic properties of the model. We simulate the model to demonstrate effects of changes in some parameters on the economic system in Section 4. Section 5 concludes the study.

2. The Basic Model

The economy has three sectors and one government (who collects taxes for environmental protection). The three sectors are production, environmental resources and renewable resource sectors. Most aspects of the production sector are similar to the production sector in the standard one-sector growth model (see Burmeister and Dobell 1970, Azariadis 1993, Barro and Sala-i-Martin 1995). It is assumed that there is only one (durable) good in the economy under consideration. Households own assets of the economy and distribute their incomes to consumption and savings. Production sectors or firms use labour and capital as inputs. Exchange takes place in a perfectly competitive markets. Saving is undertaken only by households. All earnings of firms are distributed in the form of payments to factors of production. We assume a homogenous and fixed population, N. The labour force is distributed among the three sectors. We select commodity to serve as numeraire (whose price is normalised to 1), with all the other prices being measured relative to its price. We assume that wage rates are identical among all professions.

2. 1 Production Sector

We assume that production is to combine labour force, $N_i(t)$, and physical capital, $K_i(t)$. We use the conventional production function to describe a relationship between inputs and output, except that environmental quality affects productivity. Let $F_i(t)$ stand for the output level of the production sector at time t. The production function is specified as follows:

$$F_i(t) = A_i \Gamma_i(E) K_i^{\alpha_i}(t) N_i^{\beta_i}(t), \quad A_i, \alpha_i, \beta_i > 0, \quad \alpha_i + \beta_i = 1, \tag{1}$$

where A_i, α_i, and β_i are positive parameters. Here, $\Gamma_i(E)$ is a function of the environmental quality measured by the level of pollution, $E(t)$. It is reasonable to assume that productivity is negatively related to the pollution level, that is, $\Gamma_i(E) \leq 0$.

Markets are competitive; thus labour and capital earn their marginal products. The rate of interest, $r(t)$, and wage rate, $w(t)$, are determined by markets. The marginal conditions are given by:

$$r(t) + \delta_k = \frac{\alpha_i \bar{\tau}_i F_i(t)}{K_i(t)}, \quad w(t) = \frac{\beta_i \bar{\tau}_i F_i(t)}{N_i(t)}, \tag{2}$$

where δ_k is the given depreciation rate of physical capital and τ_i is the fixed tax rate, $\bar{\tau}_i \equiv 1 - \tau_i, \ 0 < \tau_i < 1$

2.2 Change of Renewable Resources

Let $X(t)$ stand for the stock of the resource. The natural growth rate of the resource is assumed to be a logistic function of the existing stock.[1]

$$\phi_0 X(t)\left(1 - \frac{X(t)}{\phi}\right),$$

where a dot over any variable represents the change over time and the variable, ϕ is the maximum possible size for the resource stock, called the carrying capacity of the resource,

1 The logistic model, proposed early in the nineteenth century, has been frequently used in literature of growth with renewable resource (e.g., Brander and Taylor 1997; Hannesson 2000; Cairns and Tian 2010). Its wide success in different fields of biological and social sciences is its apparent empirical success.

and the variable, ϕ_0, is 'uncongested' or 'intrinsic' growth rate of the renewable resource. If the stock is equal to ϕ, then the growth rate should equal zero. If the carrying capacity is much larger than the current stock, then the growth rate per unit of the stock is approximately equal to the intrinsic growth rate. That is, the congestion effect is negligible. In this study, for simplicity, we assume both the carrying capacity and the intrinsic growth rate as constant.[2] Let $F_x(t)$ stand for the harvest rate of the resource. The change rate in the stock is then equal to the natural growth rate minus the harvest rate, that is:

$$\dot{X}(t) = \phi_0 X(t)\left(1 - \frac{X(t)}{\phi}\right) - F_x(t). \tag{3}$$

We now examine the functional form of the harvest rate. We assume a nationally owned open-access renewable resource.[3] With open access, harvesting occurs up to the point at which the current return to a representative entrant equals the entrant's cost.[4] Aside from the stock of the renewable resources, like the good sector, there are two factors of production. Let $N_x(t)$ and $K_x(t)$ respectively stand for the labour force and capital stocks employed by the resource sector. We assume that harvesting of the resource is carried out according to the following harvesting production function:

$$F_x(t) = A_x \Gamma_x(E) X^b(t) K_x^{\alpha_x}(t) N_x^{\beta_x}(t), \;\; A_x, b \ge 0, \;\; \alpha_x, \beta_x > 0, \;\; \alpha_x + \beta_x = 1, \tag{4}$$

where A_x, b, α_x and β_x are parameters and $\Gamma_i(E)$ is a function of the environmental quality measured by the level of pollution. The specified form implies that if the capital (like machine) and labour inputs are simultaneously doubled, then harvest is also doubled for a given stock of the resource at a given time. It should be noted that the Schaefer harvesting production function takes on the following form[5]:

$$F_x(t) = A_x X(t) N_x(t),$$

is evidently a special case of (4). The Schaefer production function does not take account of capital (or with capital being fixed). As machines are important inputs in harvesting, we explicitly take account of capital inputs.

Harvesting is carried out by competitive profit-maximising firms under conditions of free entry. Let $p(t)$ and τ_x respectively stand for the price of the resource and the fixed tax rate on the harvesting. We introduce $\bar{\tau}_x \equiv 1 - \tau_x$, $0 < \tau_x < 1$. The marginal conditions are given as follows:

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2 This is a strict assumption as the two variables may change due to changes in other conditions. For instance, in Jinji (2006), the carrying capacity changes as a function of the stock of a renewable resource. It is also reasonable to take account of possible effects of the environmental quality on the two parameters. For instance, as the environment deteriorates, the growth rate of the renewable resource may become lower.

3 The open-access case was initially examined by Gordon (1954).

4 This condition may not be satisfied, for instance, when property rights of the resource are incomplete.

5 See Schaefer (1957). The function with fixed capital and technology is widely applied to fishing (see also, Paterson and Wilen 1977; Milner-Gulland and Leader-Williams 1992; Bulter and van Kooten 1999).

$$r(t) + \delta_k = \frac{\alpha_x \bar{\tau}_x p(t) F_x(t)}{K_x(t)}, \quad w(t) = \frac{\beta_x \bar{\tau}_x p(t) F_x(t)}{N_x(t)}. \tag{5}$$

2.3 Environmental Change

Economic growth often implies worsened environmental conditions. Growth also implies a higher material standard of living which will, through the demand for a better environment, induce changes in the structure of the economy to improve the environment. As a society accumulates more capital and makes progresses in technology, more resources may be used to protect the environment. Tradeoffs between consumption and pollution have been extensively analysed since the publication of the seminal papers by Plourde (1972) and Forster (1973). We now describe the dynamics of the stock of pollutants, $E(t)$. We assume that pollutants are created both by production and consumption. We specify the dynamics of the stock of pollutants as follows:

$$\dot{E}(t) = \theta_i F_i(t) + \theta_x F_x(t) + \bar{\theta}_c C(t) + \bar{\theta}_x C_x(t) - Q_e(t) - \theta_0 E(t), \tag{6}$$

in which θ_i, θ_x, $\bar{\theta}_c$, $\bar{\theta}_x$, and θ_0 are positive parameters and

$$Q_e(t) = A_e \Gamma_e(E) K_e^{\alpha_{e0}}(t) N_e^{\beta_{e0}}(t), \; A_e, \alpha_{e0}, \beta_{e0} > 0, \tag{7}$$

where $N_e(t)$ and $K_e(t)$ are respectively the labour force and capital stocks employed by the environmental sector; A_e, α_e, and β_e are positive parameters, and $\Gamma_e(E)$ (≥ 0) is a function of E. The term $\theta \; F$ means that pollutants that are emitted during production processes are linearly positively proportional to the output level.[6] The parameter, $\bar{\theta}_c$, means that in consuming one unit of the good, the quantity $\bar{\theta}_c$ is left as waste.[7] The parameter θ_c depends on the technology and environmental sense of consumers. We can similarly interpret θ_x and $\bar{\theta}_x$. The parameter θ_0 is called the rate of natural purification. The term $\theta_0 E$ measures the rate that the nature purifies environment. The term, $K_e^{\alpha_e} N_e^{\beta_e}$, in Q_e means that the purification rate of the environment is positively related to capital and labour inputs. The function, $\Gamma_e(E)$ implies that the purification efficiency is dependent on the stock of pollutants. It is not easy to generally specify how the purification efficiency is related to the scale of pollutants. For simplicity, we specify Γ_e as follows $\Gamma_e(E) = \theta_e E^{\upsilon}$, where $\theta_e > 0$ and $\upsilon > 0$ are parameters. As far as economic production, capital accumulation and environmental dynamics are concerned, our model is similar to the dynamic model by Dinda (2005) in many aspects. Like in Dinda's model, we allow capital allocation between commodity production and pollution abatement; but are different from Dinda's model in which labour is omitted in the

[6] This assumption is well applied in the literature. See, for instance, Gutiérrez (2008).

[7] For instance, John and Pecchenino (1994), John *et al*. (1995) and Prieur (2009) consider that consumption degrades environment.

economy and neglect possible pollution due to consumption,[8] we allow labor allocation between commodity production and pollution abatement and explicitly treat consumption as a source of pollution.

2.4 Consumer Behaviours

We use an alternative approach to household behaviour. Rather than taking account of environmental action by firms and households, this study introduces environmental taxation on firms (outputs), wealth income, and wage income. There are models with environmental tax incidence (see, for instance, Rapanos 1992; 1995). Our approach differs from the traditional approaches also with regard to how the environmental taxation affects behaviour of households. Consumers make decisions on choice of consumption levels of the resource good and commodities as well as on how much to save. Different from the optimal growth theory in which utility defined over future consumption streams is used, we apply an alternative approach to preference structure of consumers over consumption and saving. We denote per capita wealth by $k(t)$ where $k(t) \equiv K(t)/N$. Let τ_k and τ_w respectively stand for the tax rates on the interest payment and wage income. Per capita current income from the interest payment $r(t)k(t)$ and the wage payment $w(t)$ is given by:

$$y(t) = (1 - \tau_k)r(t)k(t) + (1 - \tau_w)w(t).$$

where $\bar{\tau}_k \equiv 1 - \tau_k$ and $\bar{\tau}_w \equiv 1 - \tau_w$. We call $y(t)$ the current income. The per capita disposable income is given by:

$$\hat{y}(t) = y(t) + k(t). \tag{8}$$

The disposable income is used for saving and consumption. At each point of time, a consumer would distribute the total available budget among saving, $s(t)$, consumption of the commodity, $c(t)$, and consumption of the resource good, $c_x(t)$. The budget constraint is given by:

$$(1 + \tau_c)c(t) + s(t) + (1 + \tilde{\tau}_x)p(t)c_x(t) = \hat{y}(t), \tag{9}$$

where τ_c and are respectively the tax rates on the consumption of the goods and the resource. It should be noted that there are different taxes on households as well as producers (Bovenberg and Smulders 1995; 1996). Our approach takes account of different exogenous tax rates. It should be noted that another important issue is how to endogenously determine the tax rates.

In our model, at each point of time, consumers have four variables, $s(t)$, $c(t)$, and $c_x(t)$, to decide. For simplicity of analysis, we specify the utility function as follows:

$$U(t) = \Gamma_u(E)c^{\xi_0}(t)s^{\lambda_0}(t)c_x^{\chi_0}(t), \quad \xi_0, \lambda_0, \chi_0 > 0, \tag{10}$$

[8] In a one-sector growth model with capital and consumption goods being aggregated into a single good, the omission of consumption as a source of pollution may not affect the qualitative conclusions of the model. But when we take account of differences in taxation on production and consumption, not to mention economic structures with multiple sectors, it is important to explicitly treat consumption as a source of pollution.

where ξ_0 the propensity to consume, λ_0 the propensity to own wealth, and χ_0 the propensity to consume the resource good. A detailed explanation of the approach and its applications to different problems of economic dynamics are provided in Zhang (2005). It should be noted that this study does not explicitly take account of consumer awareness of environment. For instance, consumers may prefer environment-friendly goods when their living conditions are changed. With regard to how much money the economic agent should spend on environmental improvement, Selden and Song (1995) hold that at a lower level of pollution, the representative agent does not care much about environment and spends his resource on consumption; however, as the environment becomes worse and income becomes higher, more capital will be used for environmental improvement. We may take account of changes in consumer behaviour, for instance, by assuming that the representative consumer spends a proportion of the disposable income on environment or the tax rate on consumer consumption is explicitly related to income and consumption level.

For the representative consumer, wage rate $w(t)$ and rate of interest $r(t)$ are given in markets and wealth $k(t)$ is pre-determined before decision. Maximising $U(t)$ in (9) subject to the budget constraint (8) yields

$$c = \xi \hat{y}, \quad s = \lambda \hat{y}, \quad p c_x = \chi \hat{y}, \tag{11}$$

where

$$\xi \equiv \frac{\rho \xi_0}{1 + \tau_c}, \quad \lambda \equiv \rho \lambda_0, \quad \chi \equiv \frac{\rho \chi_0}{1 + \tilde{\tau}_x}, \quad \rho \equiv \frac{1}{\xi_0 + \lambda_0 + \chi_0}.$$

The demand for the resource is given by $c_x = \chi \bar{y} / p$. The demand decreases in its price and increases in the disposable income. An increase in the propensity to consume the resource increases the consumption when the other conditions are fixed. In this dynamic system, as any factor is related to all the other factors over time, it is difficult to see how one factor affects any other variable over time in the dynamic system. A detailed explanation of the approach and its application to different problems of economic dynamics are provided in Zhang (2005). It should be noted that in Balcao (2001) and Nakada (2004), it is assumed that utility depends negatively on pollution, which is a side product of the production process. As reviewed by Munro (2009: 43), "environmental economics has been slow to incorporate the full nature of the household into its analytical structures. … [A]n accurate understanding of household behaviour is vital for environmental economics." Our approach to household behaviour is still over-simplified as, for instance, we analyse an economy with a single good and a single pollutant. We will deal with household behaviour more realistically by, for instance, introducing multiple goods into the utility functions and each good has distinct features with regard to pollution (and may be subject to different environmental policies). We may also take into account family structure in the modeling.

We now find the dynamics of capital accumulation. According to the definition of $s(t)$, a change in the household's wealth is given by

$$\dot{k}(t) = s(t) - k(t). \tag{12}$$

The equation simply states that the change in wealth is equal to saving minus dissaving.

2.5 Capital and Labour Employed by the Environment Sector

We now determine how the government determines the number of labour force and the level of capital employed for purifying pollution. We assume that all the tax incomes are spent on environment. The government's tax incomes consist of the tax incomes on the production sector, consumption, wage income and wealth income. Hence, the government's income is given by:

$$Y_e(t) = \tau_i F_i(t) + \tau_x p(t) F_x(t) + \tau_c C(t) + \tau_x p(t) C_x(t) + \tau_w N w(t) + \tau_k r(t) K(t). \qquad (13)$$

Ono (2003) introduces tax on the producer and uses the tax income for environmental improvement in the traditional neoclassical growth theory. For simplicity, we assume that the government's income is used up only for the environmental purpose. As there are only two input factors in the environmental sector, the government budget is given by:

$$(r(t) + \delta_k) K_e(t) + w(t) N_e(t) = Y_e(t). \qquad (14)$$

We need an economic mechanism to analyse how the government distributes the tax income. We assume that the government will employ the labour force and capital stocks for purifying environment in such a way that the purification rate achieves its maximum under the given budget constraint. The government's optimal problem is given by:

$$\text{Max}\, Q_e(t) \quad \text{s.t.:}\ (r(t) + \delta_k) K_e(t) + w(t) N_e(t) = Y_e(t).$$

The optimal solution is given by:

$$(r(t) + \delta_k) K_e(t) = \alpha_e Y_e(t), \quad w(t) N_e(t) = \beta_e Y_e(t), \qquad (15)$$

where

$$\alpha_e \equiv \frac{\alpha_{e0}}{\alpha_{e0} + \beta_{e0}}, \quad \beta_e \equiv \frac{\beta_{e0}}{\alpha_{e0} + \beta_{e0}}.$$

2.6 Demand for and Supply of the Resource

The demand for and supply of the resource balance at any point of time

$$c_x(t) N = F_x(t). \qquad (16)$$

Let N and $K(t)$ stand for respectively the labour supply and total capital stock. The labour force is allocated between the three sectors. As full employment of labour and capital is assumed, we have

$$K_i(t) + K_x(t) + K_e(t) = K(t), \quad N_i(t) + N_x(t) + N_e(t) = N. \qquad (17)$$

We have thus built the dynamic model. We now examine dynamics of the model.

3. The Dynamics and its Properties

First, we introduce a new variable by $z(t) \equiv (r(t) + \delta_k)/w(t)$. We now show that the dynamics can be expressed by the 3-dimensional differential equations system with $z(t)$, $X(t)$ and $E(t)$ as variables.

Lemma 1

The motion of the system is determined by the 3-dimensional differential equations

$$\begin{aligned} \dot{z} &= \overline{\Lambda}_z(z, X, E), \\ \dot{X} &= \overline{\Lambda}_X(z, X, E), \\ \dot{E} &= \overline{\Lambda}_E(z, X, E), \end{aligned} \tag{18}$$

where the functions in (18) are functions of $z(t)$, $X(t)$ and $E(t)$ given in the appendix. Moreover, all the other variables can be determined as functions of $z(t)$, $X(t)$ and $E(t)$ at any point of time by the following procedure: K by (A18) $\rightarrow K_i$ and K_x by (A14) $\rightarrow K_e$ by (A12) $\rightarrow N_i$, N_x and N_e by (A1) $\rightarrow F_x$ by (4) $\rightarrow F_i$ by (1) $\rightarrow Q_e$ by (7) $\rightarrow p$, r and w by (2) $\rightarrow k = K/N \rightarrow \hat{y}$ by (8) $\rightarrow c$, c_x and s by (11).

Lemma 1 shows that once we determine the values of the three variables with some initial conditions, we can determine all the variables in the economic system. The lemma is important as it provides a procedure to follow the motion of the system with computer with a given initial condition. The differential equations system (19) contains three variables. A steady state is determined by

$$\begin{aligned} \overline{\Lambda}_z(z, X, E) &= 0, \\ \overline{\Lambda}_X(z, X, E) &= 0, \\ \overline{\Lambda}_E(z, X, E) &= 0. \end{aligned} \tag{19}$$

As the expressions of the analytical results are tedious, for illustration, we specify the parameter values as follows:

$$\begin{aligned} &N_0 = 5,\ \alpha_i = 0.32,\ A_i = 1,\ \alpha_x = 0.4,\ A_x = 0.7,\ \alpha_{e0} = 0.6,\ A_e = 0.3,\ \phi = 3,\ \phi_0 = 5, \\ &\tau_i = 0.02,\ \varepsilon_i = 0.6,\ \tau_x = 0.04,\ \tau_k = 0.01,\ \tau_w = 0.01,\ \tau_c = 0.04,\ \lambda_0 = 0.6,\ \xi_0 = 0.09, \\ &\chi_0 = 0.02,\ b = 0.7,\ b_i = 0.3,\ b_e = 0.2,\ b_x = 0.2,\ \theta_i = 0.03,\ \theta_x = 0.03,\ \overline{\theta}_c = 0.03, \\ &\overline{\theta}_x = 0.02,\ \theta_0 = 0.04,\ \delta_k = 0.05. \end{aligned} \tag{20}$$

The population is 5. The adjustment speed, ϕ, is fixed at 3. We assume that the propensity to save is much higher than the propensity to consume the commodity and the propensity to consume the renewable resource. Some empirical studies on the US economy demonstrate that the value of the parameter, a, in the Cobb-Douglas production is approximately equal 0.3 to (for instance, Miles and Scott 2005; Abel *et al.* 2007). With regard to the technological parameters, important to our study are their relative values. The tax rates are all assumed low. Under (20), we identify the equilibrium values as follows:

$$\begin{aligned} &K = 32.45,\ E = 1.69,\ X = 2.61,\ F_i = 6.31,\ F_x = 1.71,\ Q_e = 0.35,\ N_i = 4.22, \\ &N_x = 0.60,\ N_e = 0.18,\ K_i = 24.22,\ K_x = 4.89,\ K_e = 3.34,\ p = 0.61,\ r = 0.032, \\ &w = 1.00,\ c_X = 0.34,\ c = 0.94,\ s = 6.49. \end{aligned} \tag{21}$$

The three eigenvalues are calculated as

$$\{-4.146, -0.188, -0.092\}.$$

We see that the dynamic system has a unique stable equilibrium. Hence, for any initial state, the system should move toward the equilibrium. With the initial conditions, we plot the motion of the economic system as in Figure 1. We see that the economy approaches its equilibrium over time.

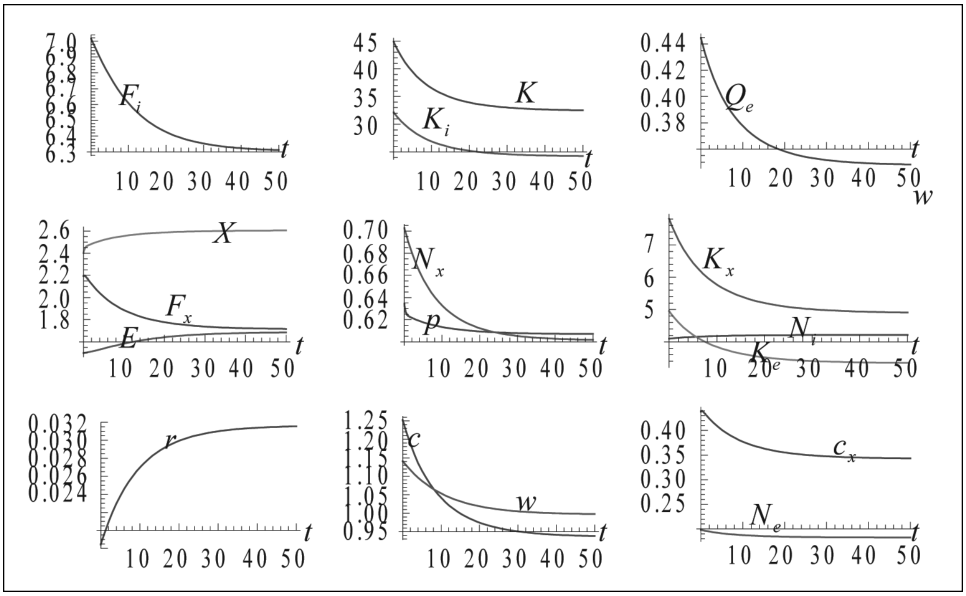

Figure 1. *Motion of the economic system*

4. Comparative Dynamic Analysis

This section examines effects of changes in some parameters on the motion of the economic system. First, we study the case that all the parameters, except the population, are the same as in (20). We increase the population in the following way: $N_0 = 5 \Rightarrow 5.2$. The simulation results are demonstrated in Figure 2. In the plots, a variable $\overline{\Delta} x_j(t)$ stands for the change rate of the variable $x_j(t)$ in percentage due to changes in the parameter value. We will use the symbol $\overline{\Delta}$ with the same meaning when we analyse other parameters. The rise in the population increases the total output of the industrial sector. As the total population is increased, the labour inputs of all the sectors are increased. Also as a consequence of the rise in the population, the total capital and capital inputs of all the sectors are increased. Hence, although the environment deteriorates and the productivity of the production sector falls, the output level of the production sector is increased as the inputs of capital and labour are increased. In association with the rise in the total capital, the rate of interest is increased. The total resource is reduced and the price of the resource is increased. The output levels of the resource sector and environmental sector are increased. The wage rate is reduced. The consumption of the goods and the resource per capita are reduced. As the environment is deteriorated, and the wealth per capita and the consumption levels are reduced, we see that each household suffers from the population increase. This negative impact occurs as the economic system is typically 'neoclassical'. If we introduce increasing returns and endogenous knowledge, our conclusion on impact of population change on the household may be different.

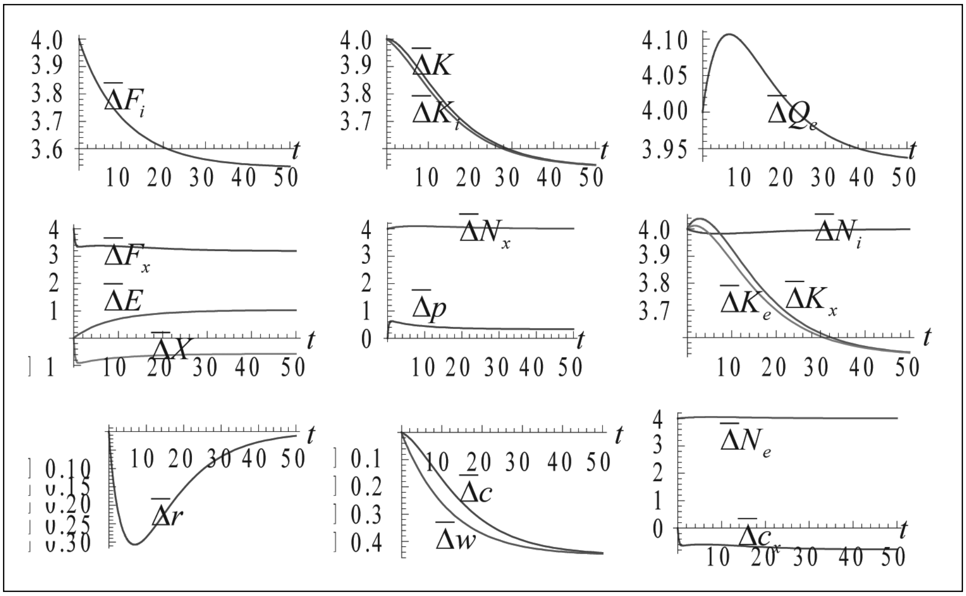

Figure 2. *A rise in the population*

We now study the case that the propensity for consuming the resource is increased in the following way: $\chi_0 = 0.02 \Rightarrow 0.025$. The simulation results are plotted in Figure 3. As the preference for the resource is strengthened, the price of the resource is increased and the resource production is increased. The stock of the resource is reduced. More labour force is employed by the resource and environmental sectors. The total capital rises initially but falls in the long term. The capital input employed by the resource sector is increased; the capital input employed by the production sector is reduced. This occurs as the rise in the propensity to consume resources increases the demand for the resource and thus increases the price. The rise in the price makes the resource sector (with the other conditions fixed) to attract more capital and labour. As the labour force is fixed, the labour force employed by the production sector is reduced. The environment sector also absorbs more labour. The output for environmental protection rises initially but falls in the long term. This occurs partly because the capital input of the sector falls in the long term. The rate of interest rises in association with the fall in the total capital. The wage rate falls. The consumption level of the resource is increased but that of the good is reduced. The stock of resource falls, even though more resources are employed by the resource sector. The pollutant level falls slightly but rises in the long term.

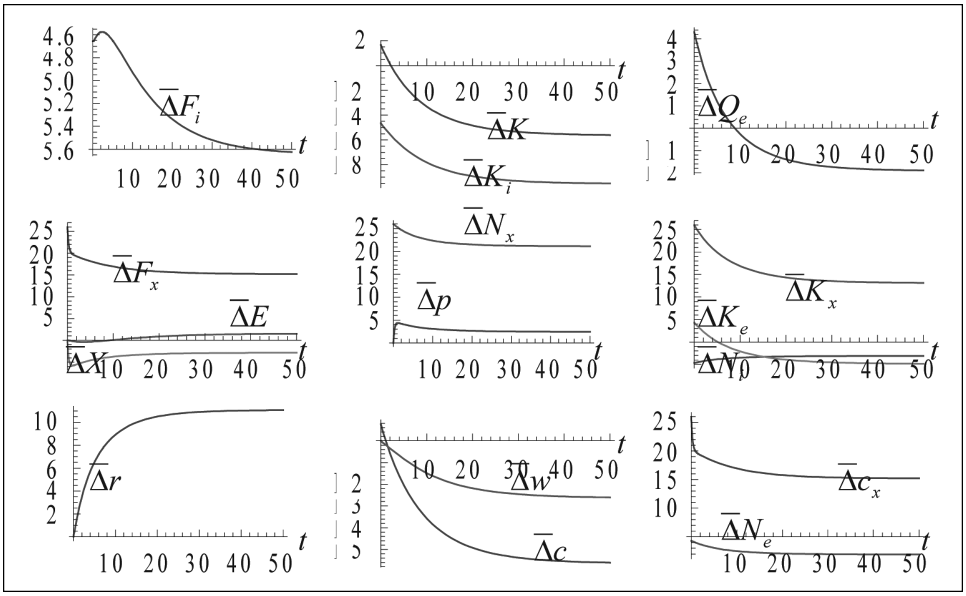

Figure 3. *A rise in the propensity to consume the resource*

We now study the case that the government increases taxes on the household's consumption of the good. Tax is increased in the following way: $\tau_c = 0.04 \Rightarrow 0.06$ The simulation results are plotted in Figure 4. As the tax rate on the consumption of the good is increased, the level of the consumption is reduced initially, but increased soon over time. If the model does not include endogenous resources and environment, it is reasonable to expect that a rise in the tax rate on the good would reduce consumption level of the good. Nevertheless, we observe that the consumption rises instead falls. The dynamic mechanism of the effect is partly explained as follows. As the rate is increased, the consumption level falls immediately. As the tax income from taxing the consumer is used for the environment, the efforts towards improving the environment are increased both in terms of capital and labour inputs. This results in environmental improvement, which enhances the productivity of goods production. Hence, the supply tends to be increased (with the other conditions fixed) because of the environmental improvement. As more goods are produced, the relative price of the good tends to fall. This is reflected in the rise of the resource price. As the price of the good falls relatively and the household's income from holding wealth and wage is increased, the household consumption level of the good is increased even though the household has to pay more tax on each unit of the good consumed. We also see that the total capital and the capital input of the environment sector are increased over time. The capital inputs of the other two sectors fall initially but rise soon. The resource stock is reduced and its price is increased over time. Resource consumption is increased over time. The rate of interest and wage rate rise over time. The labour force of the resource sector rises initially, then falls, and rises again. The labour force of the environmental sector is increased over time and the labour force of the production sector is reduced.

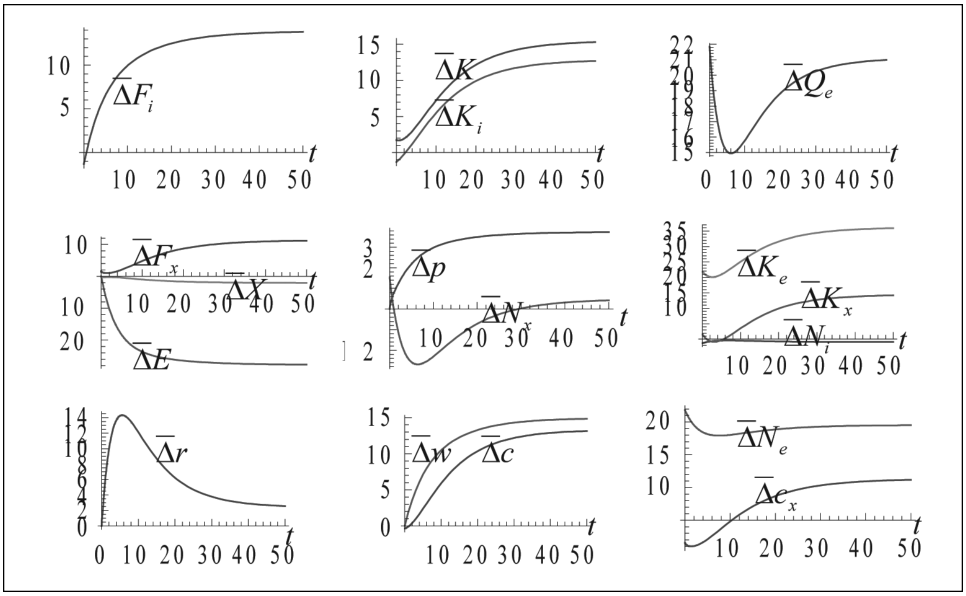

Figure 4. *A rise in the tax on consumption of goods*

We now examine a case that the resource capacity is increased as follows: $\phi = 3 \Rightarrow 3.5$. We see that as the capacity is increased, the stock of the resource is increased. In association with the increase in the resource stock, the price of the resource is reduced. The output level of the environmental sector rises initially and falls in the long term. The resource consumption is increased and the resource price is reduced. The rate of interest falls initially and is almost not affected in the long term. The consumption level of the good and the wage rate fall over time. The total capital and capital inputs to the three sectors are all reduced in the long term.

5. Concluding Remarks

The model in this study describes a dynamic interdependence among physical accumulation, environmental change, resource change and division of labor under perfect competition with environmental taxes on production, wealth income, wage income and consumption. We synthesised the growth mechanism in the neoclassical growth theory, the environmental dynamics in traditional models of environmental economics, and resource dynamics and utilisation in resource economics within a comprehensive framework with an alternative approach to household behaviour. We simulated the model to demonstrate existence of equilibrium points and motion of the dynamic system. The simulation demonstrates some dynamics which can be predicted neither by the neoclassical growth theory nor by the traditional economic models of environmental change and resource economics. We may extend the model in some directions. For instance, we may introduce leisure time as an endogenous variable. Munro (2009: 3) correctly points out: "In the unitary model, the household acts as if it is a single individual maximising a single utility function in the face of

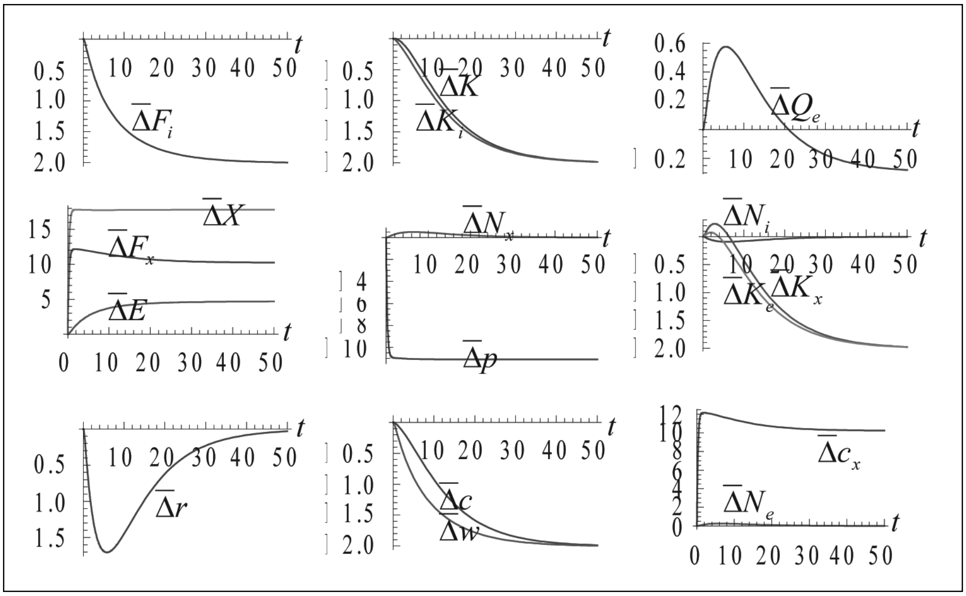

Figure 5. *A rise in the resource capacity*

one budget constraint. It is a simplifying modeling assumption that is widely used in most branches of economics, but it is wrong. The fact that the unitary model is inaccurate is well-known and has been known for many years now." It is important to take account of family structure as well as economic structure in analysing relations among growth, resource dynamics and environmental change. As demonstrated by Hamilton and Zilberman (2006), consumers voluntarily pay significant price premiums to acquire environmental attributes in environment-friendly products. Whether fast economic growth will hurt or improve environmental quality is also dependent on the pollutant.[9] We may analyse this issue by introducing multiple goods into the model. Another important extension of this research is to study dynamic interdependence among economic growth, health and environment.[10]

References

Abel, A., B.S. Bernanke and D. Croushore. 2007. *Macroeconomics*. New Jersey: Prentice Hall.

Azariadis, C. 1993. *Intertemporal Macroeconomics*. Oxford: Blackwell.

Balcao, A. 2001. Endogenous growth and the possibility of eliminating pollution. *Journal of Environmental Economics and Management* **42**: 360-373.

Barro, R.J. and X. Sala-i-Martin 1995. *Economic Growth*. New York: McGraw-Hill, Inc.

Bovenberg, A.L. and S. Smulders. 1995. Environmental quality and pollution-augmenting technical change in a two-sector endogenous growth model. *Journal of Public Economics* **57**: 369-391.

[9] See, for instance, Dinda (2004).

[10] There are already some studies on these issues by, for instance, Williams (2002), Chakraborty (2004), Tang and Zhang (2007), and Gutiérrez (2008).

Bovenberg, A.L. and S. Smulders. 1996. Transitional impacts of environmental policy in an endogenous growth model. *International Economic Review* **37**: 861-893.

Brander, J.A. and M.S. Taylor. 1998. The simple economics of Easter Island: A Ricardo-Malthus model of renewable resource use. *American Economic Review* **81**: 119-138.

Bulter, E.H. and G.C. Van Kooten. 1999. Economics of anti-poaching enforcement and the ivory trade ban. *American Journal of Agricultural Economics* **81**: 453-466.

Burmeister, E. and A.R. Dobell. 1970. *Mathematical Theories of Economic Growth*. London: Collier Macmillan Publishers.

Cairns, D.R. and H.L. Tian. 2010. Sustained development of a society with a renewable resource. *Journal of Economic Dynamics & Control* **24**: 2048-2061.

Chakraborty, S. 2004. Endogenous lifetime and economic growth. *Journal of Economic Theory* **116**: 119-137.

Dinda, S. 2004. Environmental Kuznets curve hypothesis: A survey. *Ecological Economics* **49**: 431-455.

Dinda, S. 2005. A theoretical basis for the environmental Kuznets curve: Turning point and public policy. *Environmental and Resource Economics* **36**: 15-34.

Forster, B.A. 1973. Optimal consumption planning in a polluted environment. *Economic Record* **49**: 534-545.

Fullerton, D. and S.R. Kim. 2008. Environmental investment and policy with distortionary taxes, and endogenous growth. *Journal of Environmental Economics and Management* **56**: 141-154.

Gordon, H.S. 1954. The economic theory of a common property resource: The fishery. *Journal of Political Economy* **62**: 124-142.

Gutiérrez, M. 2008. Dynamic inefficiency in an overlapping generation economy with pollution and health costs. *Journal of Public Economic Theory* **10**: 563-594.

Hamilton, S.F. and D. Zilberman. 2006. Green markets, eco-certification, and equilibrium fraud. *Journal of Environmental Economics and Management* **52**: 627-644.

Hannesson, R. 2000. Renewable resources and the gains from trade. *Canadian Journal of Economics* **33**: 122-132.

Jinji, N. 2006. International trade and terrestrial open-access renewable resources in a small open economy. *Canadian Journal of Economics* **39**: 790-808.

John, A. and R. Pecchenino. 1994. An overlapping generation model of growth and the environment. *The Economic Journal* **104**: 1393-1410.

John, A., Pecchenino, R., Schimmelpfennig, D. and S. Schreft. 1995. Short-lived agents and the long-lived environment. *Journal of Public Economics* **58**: 124-141.

Kijima, M., K. Nishide and A. Ohyama. 2010. Economic models for the environmental Kuznets curve: A survey. *Journal of Economic Dynamics & Control* **34**: 1187-1201.

Miles, D. and A.D. Scott. 2005. *Macroeconomics – Understanding the WEALTH of Nations*. Chichester: John Wiley & Sons, Ltd.

Milner-Gulland, E.J. and N. Leader-Williams. 1992. A model of incentives for the illegal exploitation of black rhinos and elephants. *Journal of Applied Ecology* **29**: 388-401.

Munro, A. 2009. Introduction to the special issue: Things we do and don't understand about the household and the environment. *Environmental and Resources Economics* **43**: 1-10.

Munro, G.R. and A.D. Scott. 1985. The economics of fisheries management. In *Handbook of Natural Resource and Energy Economics*, Vol. II, ed. A.V. Kneese and J.L. Sweeney. Amsterdam: Elsevier.

Nakada, M. 2004. Does environmental policy necessarily discourage growth? *Journal of Economics* **81**: 249-268.

Ono, T. 2003. Environmental tax policy and long-run economic growth. *The Japanese Economic Review* **53**: 202-217.

Paterson, D.G. and J.E. Wilen. 1977. Depletion and diplomacy: The North-Pacific seal hunt, 1880-1910. In *Research in Economic History*, ed. P. Uselding. Greenwich: JAI Press.

Plourde, G.C. 1972. A model of waste accumulation and disposal. *Canadian Journal of Economics* **5**: 119-125.
Prieur, F. 2009. The environmental Kuznets curve in a world of irreversibility. *Economic Theory* **40**: 57-90.
Rapanos, V.T. 1992. A note on externalities and taxation. *Canadian Journal of Economics* **25**: 226-232.
Rapanos, V.T. 1995. The effects of environmental taxes on income distribution. *European Journal of Political Economy* **11**: 487-501.
Schaefer, M.B. 1957. Some considerations of population dynamics and economics in relation to the management of marine fisheries. *Journal of Fisheries Research Board of Canada* **14**: 669-681.
Selden, T.M. and D. Song. 1995. Neoclassical growth, the J curve for abatement and the inverted U curve for pollution. *Journal of Environmental Economics and Management* **29**: 162-168.
Stiglitz, J.E. 1974. Growth with exhaustible natural resources: Efficient and optimal growth paths. *Review of Economic Studies* **41**: 123-137.
Solow, R. 1999. Neoclassical growth theory. In *Handbook of Macroeconomics*, ed. J.B. Taylor, and M. Woodford. Amsterdam: North-Holland.
Tang, K.K. and J. Zhang. 2007. Health, education and life-cycle savings in the development process. *Economic Inquiry* **45**: 615-630.
Tsurumi, T. and S. Managi. 2010. Decomposition of the environmental Kuznets curve: Scale, technique, and composition effects. *Environmental Economics and Policy Studies* **11**: 19-36.
Williams, R.C.III. 2002. Environmental tax interaction when pollution affects health or productivity. *Journal of Environmental Economics and Management* **87**: 271-70.
Zhang, W.B. 2005. *Economic Growth Theory*. London: Ashgate.
Zhang, W.B. 2011. Environmental policy and pollution dynamics in an economic growth model. *Modern Economy* **2**: 233-41.
Zhang, W.B. 2011a. Renewable resources, capital accumulation, and economic growth. *Business Systems Research* **1**: 24-35.

Appendix: Proving Lemma 1

The appendix shows that the dynamics can be expressed by three-dimensional differential equations. From (2) and (5), and (15), we obtain

$$z \equiv \frac{r + \delta_k}{w} = \frac{\tilde{\alpha}_i N_i}{K_i} = \frac{\tilde{\alpha}_x N_x}{K_x} = \frac{\tilde{\alpha}_e N_e}{K_e}, \tag{A1}$$

where we omit time index and $\tilde{\alpha}_j \equiv \alpha_j / \beta_j, \ j = i, x, e$. Insert (A1) in $N_i + N_x + N_e = N$ in (18)

$$\frac{K_i}{\tilde{\alpha}_i} + \frac{K_x}{\tilde{\alpha}_x} + \frac{K_e}{\tilde{\alpha}_e} = \frac{N}{z}. \tag{A2}$$

Insert (A1) and (1) in (2)

$$r + \delta_k = \frac{\alpha_i \bar{\tau}_i A_i \Gamma_i z^{\beta_i}}{\tilde{\alpha}_i^{\beta_i}}, \quad w = \frac{\beta_i \tilde{\alpha}_i^{\alpha_i} \bar{\tau}_i A_i \Gamma_i}{z^{\alpha_i}}. \tag{A3}$$

From the definition of Y_e we have

$$Y_e = \tau_i F_i + \tau_x p_x F_x + \tau_0 N \hat{y} + \tau_w N w + \tau_k r K, \tag{A6}$$

where we use (11) and $\tau_0 \equiv \tau_c \xi + \tau_x \chi$. Insert (8) in (A6)

$$Y_e = \frac{\tau_i w N_i}{\beta_i \bar{\tau}_i} + \frac{\tau_x w N_x}{\beta_x \bar{\tau}_x} + \tau_0 K + (\bar{\tau}_k \tau_0 + \tau_k) r K + (\bar{\tau}_w \tau_0 + \tau_w) N w, \tag{A7}$$

where we also use (2) and (5). From (A7), $w N_e = \beta_e Y_e$ and $z = (r + \delta_k)/w$ we have

$$\frac{N_e}{\beta_e} = \frac{\tau_i N_i}{\beta_i \bar{\tau}_i} + \frac{\tau_x N_x}{\beta_x \bar{\tau}_x} + \frac{\bar{\tau}_0 K}{w} + h(z, K), \tag{A8}$$

where

$$\bar{\tau}_0 \equiv \tau_0 - (\bar{\tau}_k \tau_0 + \tau_k)\delta_k, \ \ h(z, K) \equiv (\bar{\tau}_k \tau_0 + \tau_k) z K + (\bar{\tau}_w \tau_0 + \tau_w) N.$$

Substituting $p c_x = \chi \hat{y}$ in (11) into (16) yields

$$\chi N \hat{y} = p F_x, \tag{A9}$$

where we also use (2). Insert (8) in (A9)

$$(1 + \bar{\tau}_k r) \chi K + \bar{\tau}_w w \chi N = \frac{w N_x}{\beta_x \bar{\tau}_x}, \tag{A10}$$

where we also use (2) and (5). From (A10) and $z = (r + \delta_k)/w$, we have

$$\frac{K}{w} + \bar{\gamma}_k \bar{\tau}_k z K + \bar{\gamma}_k \bar{\tau}_w N = \frac{\bar{\gamma}_k N_x}{\chi \beta_x \bar{\tau}_x}, \tag{A11}$$

where $\bar{\gamma}_k \equiv 1/(1 - \bar{\tau}_k \delta_k)$. From (A8) and (A11), we solve

$$K_e = h_i K_i + h_x K_x + \bar{h}, \tag{A12}$$

where we use (A1) and

$$h_i \equiv \frac{\tau_i}{\bar{\tau}_i}\frac{\tilde{\alpha}_e \beta_e}{\tilde{\alpha}_i \beta_i}, \quad h_x \equiv \left(\tau_x + \frac{\bar{\tau}_0 \bar{\gamma}_k}{\chi}\right)\frac{\tilde{\alpha}_e \beta_e}{\tilde{\alpha}_x \bar{\tau}_x \beta_x}, \quad \bar{h}(z, K) \equiv \frac{\bar{h}_0}{z} + \bar{h}_k K,$$

$$\bar{h}_0 \equiv \left(\bar{\tau}_w \tau_0 + \tau_w - \bar{\tau}_0 \bar{\gamma}_k \bar{\tau}_w\right)\tilde{\alpha}_e \beta_e N, \quad \bar{h}_k \equiv \left(\bar{\tau}_k \tau_0 - \bar{\gamma}_k \bar{\tau}_0 \bar{\tau}_k + \tau_k\right)\tilde{\alpha}_e \beta_e .$$

Substituting (A12) into (A2) and $K_i + K_x + K_e = K$, we get

$$(1 + h_i)K_i + (1 + h_x)K_x = K - \bar{h},$$
$$\tilde{h}_i K_i + \tilde{h}_x K_x = \tilde{h}, \tag{A13}$$

where

$$\tilde{h}_i \equiv \frac{1}{\tilde{\alpha}_i} + \frac{h_i}{\tilde{\alpha}_e}, \quad \tilde{h}_x \equiv \frac{1}{\tilde{\alpha}_x} + \frac{h_x}{\tilde{\alpha}_e}, \quad \tilde{h}(z, K) \equiv \left(N - \frac{\bar{h}_0}{\tilde{\alpha}_e}\right)\frac{1}{z} - \frac{\bar{h}_k K}{\tilde{\alpha}_e}.$$

Solve (A13)

$$K_i = d_{ik} K - \frac{d_{iz}}{z},$$
$$K_x = d_{xk} K + \frac{d_{xz}}{z}, \tag{A14}$$

where

$$d_{ik} \equiv \left\{\tilde{h}_x + \frac{(1 + h_x)\bar{h}_k}{\tilde{\alpha}_e} - \tilde{h}_x \bar{h}_k\right\}D, \quad d_{iz} \equiv \left\{\tilde{h}_x \bar{h}_0 + (1 + h_x)\left(N - \frac{\bar{h}_0}{\tilde{\alpha}_e}\right)\right\}D,$$

$$d_{xk} \equiv \left\{\tilde{h}_i \bar{h}_k - \frac{(1 + h_i)\bar{h}_k}{\tilde{\alpha}_e} - \tilde{h}_i\right\}D, \quad d_{xz} \equiv \left\{(1 + h_i)\left(N - \frac{\bar{h}_0}{\tilde{\alpha}_e}\right) + \tilde{h}_i \bar{h}_0\right\}D,$$

$$D \equiv \frac{1}{(1 + h_i)\tilde{h}_x - (1 + h_x)\tilde{h}_i}.$$

From (A14), K_i and K_i are functions of z and By (A12) and (A1), we have K_e, N_i, N_x and N_e as functions of *z* and *K*. By (A5) K_e is a function of *z*, *K*, *E* and From (A3) we determine *r* and *w* as functions of *z* and *E*. We can also solve the following variables as functions of *z*, *K*, *E* and *X* : F_i by (1), F_x by (4), Q_e by (7), *p* by (5), $\hat{y}$ by (8), and c_x, *c* and *s* by (11).

From (2) and (5)

$$pF_x = \frac{\beta_i \bar{\tau}_i N_x F_i}{\beta_x \bar{\tau}_x N_i}. \tag{A15}$$

Insert (A15) in (A9)

$$\left(1+\bar{\tau}_k r\right)K+\bar{\tau}_w N w = \frac{\beta_i \bar{\tau}_i N_x}{\beta_x \bar{\tau}_x N_i}\frac{F_i}{\chi}, \tag{A16}$$

where we also use $y=\left(1+\bar{\tau}_k r\right)k+\bar{\tau}_w w$. From (1) and (A1), we have

$$F_i = \frac{\tilde{\alpha}_i^{\alpha_i} A_i \Gamma_i N_i}{z^{\alpha_i}}. \tag{A17}$$

Insert (A17) in (A16)

$$\left(1+\bar{\tau}_k r\right)K+\bar{\tau}_w N w = \frac{w z K_x}{\tilde{\alpha}_x \chi \beta_x \bar{\tau}_x}, . \tag{A18}$$

where we also use (A1) and (A3). Inserting (A14) in (A18), we solve K as a function of z and E as follows

$$K=\Phi(z, E)\equiv\left(\frac{d_{xz}}{\tilde{\alpha}_x \chi \beta_x \bar{\tau}_x}-\bar{\tau}_w N\right)\left(1+\bar{\tau}_k r-\frac{d_{xk} w z}{\tilde{\alpha}_x \chi \beta_x \bar{\tau}_x}\right)^{-1} w. \tag{A19}$$

From (A18), we uniquely determine K as a function of z and E. Hence, any variable can be determined as functions of z, E and X at any point of time. From (3) and (6), we have

$$\dot{X}(t)=\bar{\Lambda}_X(z, X, E),$$
$$\dot{E}(t)=\bar{\Lambda}_E(z, X, E), \tag{A20}$$

where we do not give expressions of $\bar{\Lambda}_X$ and $\bar{\Lambda}_X$ as the expressions are tedious.

Taking derivatives of (A19) with respect to t yields

$$\dot{K}=\frac{\partial\Phi}{\partial z}\dot{z}+\frac{\partial\Phi}{\partial E}\bar{\Lambda}_E. \tag{A21}$$

where we do not give expressions of $d\Phi/dz$ as the expression is tedious. Multiplying the two sides of (12) with N and using $s=\lambda\hat{y}$, we obtain

$$\dot{K}=\lambda N \hat{y}(z, X, E)-\Phi(z, E). \tag{A22}$$

From (A21) and (A22), we solve

$$\dot{z}=\bar{\Lambda}_z(z, X, E)\equiv\left[\lambda N \hat{y}(z, X, E)-\Phi(z, E)-\frac{\partial\Phi}{\partial E}\bar{\Lambda}_E\right]\left(\frac{\partial\Phi}{\partial z}\right)^{-1}. \tag{A23}$$

We have thus proved Lemma 1.