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Mathematical Modelling of Population Dynamics with Growth Sensitivity of Abuja Federal Territory, Nigeria

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RESEARCH ARTICLE

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Abstract

Keeping track of the human population is essential for proper planning for facilities such as healthcare, infrastructure, education, and other essential needs. There are various ways by which the government can ensure that service provision is improved and maintained for its citizens and very often this starts by knowing the changes in demography as a function of time. In this work, mathematical modeling and simulations are used to study the population dynamics of Abuja. The models are used for prediction of the population and how its dynamics change over time. With approximate growth rate at 9.3% per annum, the projected population of Abuja will hit 30,220,701 million by the year 2039 all things being equal. Parameter sensitivity analysis was performed using population census data, and the results show a huge influence of variations in the model parameters. The results indicate that the difference between the per capita birth and death rate parameters is crucial for changes in the population. Such findings can also be analogously applied to other cities with a similar population structure and economy.

Keywords: Birth Rate, Death Rate, Population Growth, Parameter Sensitivity Analysis

1. Introduction

Nigeria is one of the fastest growing countries in the world, with an estimated population of over 180 million and an annual population growth rate of 2.9%. Nigeria is the most populous black nation in Sub-Sahara Africa and the tenth most populous in the world according to National Population Commission, see National Bureau, 2017; National Population, 2009; Ekakitie and Ekereke, 2019. Already, the provision of essential social and basic needs services such as water, housing, education, and hospitals is a challenge not only to developed countries but even more so to developing countries such as Nigeria. It is often poorly understood how changes in certain crucial parameters influence the total population of people. Gaining insight into how small and large variations in parameters impact the national population is useful not only for government planning but also for ensuring that in the event of a sudden outbreak of epidemics, communities are well prepared to cope. One of the tools that have been effectively used in tackling such challenges is mathematical modeling and simulation (see Akhmetzhanov et al., 2019; Dzerzhinsky et al., 2021; Manu and Shikaa, 2023). However, this process requires good quality data collected from population censuses. Such data acquisition processes are very expensive and time-consuming. Many third-world countries are not equipped both financially and technologically to cope with such challenges alongside the already existing ones. Therefore, by using existing data from previous studies or information available from government archives, mathematical modeling and simulations can be used to make predictions about the population in the future and the associated implications of such predictions on the daily lives of the people (as demonstrated and discussed in previous studies). We live in a universe of limited resources, and having a population that nearly doubles every two decades puts Abuja on a trajectory of unsustainability. With the ever-increasing demand for food, water, and fuels, it has become crucial for every community to plan for its population. Proper planning will ensure that limited resources are not depleted, but rather sustained within the next few decades and beyond. In this paper, mathematical modeling and simulations are performed using prior knowledge of parameters that are considered crucial for understanding the population dynamics of Abuja. In the literature, little attention has been devoted to unraveling the complexity of the dynamics of population growth models for a city such as Abuja. In a city like Abuja, the adequate provision of many basic needs such as shelter, water, and other socio-economic needs lags behind. There is also a problem of insufficient communal facilities such as public toilets or latrines for hygiene, good sanitation, infrastructure, and general welfare. Adequate provision for most if not all of these elements relies heavily on the government having up-to-date knowledge not only on the number of people living in Abuja but also on those that immigrate, emigrate, or die. First, an analytic assessment of the model is given. This is followed by an assessment of the implications of the results from the computer simulation. Already, the Abuja's population has nearly doubled in just about two decades. Such rapid population growth can be argued to be essential for the Nigeria’s economic growth. However, it remains a challenge for Abuja to cope with its high birth rate, which by the year 2006 was reported to be about 3.1%, with a total birth rate of about 6 to 7 children per woman. Recently, a national gazette published by the National Bureau of Statistics in 2017, highlighted the potential benefits and challenges that might arise from the young and rapidly growing population in Abuja. The gazette noted that a total of 78% of residents were below the age of 30 years, and 52% were below the age of 15 years. They also observed that 6.5 million in the age group 18-30 years constituted 21.3% of the national population, an age group that was projected to grow to about 21.6 million by 2050 (see Akhmetzhanov et al., 2019; Figueroa et al., 2020; Giaimo et al., 2022). Clearly, this kind of demography is worrying, and much remains to be done by the relevant government authorities charged with planning, allocation, and management of national resources. It would be naive to assume that such demography and associated problems are limited to Abuja. The findings and recommendations from this study would therefore be applicable to states with similar demography and challenges as those of Abuja, e.g., Lagos, Rivers, Kano, Anambra, Kogi, and Sokoto, among others. Abuja, is the capital of Nigeria and one of the fastest-growing cities in the world, with a projected population of over 30.2 million in 2039 and an annual growth rate of 9.3%.

1.1 Study Area

Geographically, Abuja is located at coordinates $9^{\circ}04'N$ and $7^{\circ}29E$, covering an area of 570 square miles. It lies north of the confluence of the Niger and Benue rivers and is bordered by Niger to the west and north, Kaduna to the northeast, Nasarawa to the east and south, and Kogi to the southwest. The indigenous tribes in Abuja include Koro, Gbari, Gade, Nupe, Gwandara, Dibo, Bassa, Ganagana, and Ebira. Settler groups such as the Hausa, Igbo, and Yoruba also make up the population. Abuja is currently divided into six local government areas: Abaji, Abuja Municipal, Bwari, Gwagwalada, Kuje, and Kwali. Historically, Abuja was a center for extensive trade before British colonization, with products ranging from agricultural goods like sheanut oil, honey, and kola nuts to manufactured goods such as textiles, leather, and pottery. Abuja was carved out from Niger, Plateau, and Kwara states in 1976 (see Caswell et al., 2019; Ingiabuna et al., 2016; Manna et al., 2024).

2. Materials and Methods

Many models have been proposed in the literature as state-of-the-art approaches for modeling population growth. These models include the Exponential population growth model by Malthus, the Logistic growth model, which is a modified version of Malthus' model, and the Gompertz population growth model. Details of the theory and underlying principles are not discussed here since they can be found in the literature. Many variants of these models have been useful in predicting various populations, including fish, human, and microbial populations. These predictions are necessary for monitoring populations and ensuring the protection of specific species of animals, for instance, in national parks or large water reserves where fishing is not allowed. However, the need to use mathematical models to predict population growth is crucial for every society and nation, especially those with rapidly changing demography in third-world countries. In this work, an illustration of how mathematical models can be used to identify sensitive parameters is demonstrated. The Logistic population growth model has long been used for predicting and studying the population dynamics of different species of animals. In some cases, these models are applied to specific populations. Consider the population growth model as described by Akaligwo et al. (2024); Marrec et al., (2023); Caswell et al., (2019) which is given as:

$$\frac{dN(t)}{dt} = rN(t)\left(1 - \frac{N(t)}{K}\right) \qquad N(0) > 0 \tag{2.1}$$

where $N(t)$ is the population at any given time t, and r and K are vital parameters of the population. Here, r is often referred to as the Malthusian parameter, representing the difference between the per-capita birth and death rate in a population, and K is the carrying capacity of the population. This logistic model is a modification of the Malthusian model, which assumes exponential growth. The logistic model is more realistic compared to Malthus' exponential model, where it was assumed that the population grows at a rate proportional to the current population size, without accounting for factors like limited resources or overcrowding. In contrast, the logistic model incorporates a "crowding" effect that slows growth as the population approaches its carrying capacity (see Abu et al., 2018; Nasir et al., 2020; Marrec et al., 2023; Wali et al., 2012). The differential equation above has the solution:

$$N(t) = \frac{N(0)K}{N(0)+(K-N(0))e^{-rt}} \tag{2.2}$$

This expression can be used to estimate the population at a given time, provided that information about the parameters and initial conditions is available. In the limit as $t \to \infty$, the population approaches the carrying capacity K. At steady state, the population remains constant.

2.1 Population Growth Model

At steady state, the population $N(t)$ approaches the carrying capacity K, i.e.,

$$N(\infty) = K \quad \text{for } r < 0$$

At this point, the rate of population growth becomes zero. This implies that for very large populations, the environment's resources are fully utilized, and no further growth can occur. The population is thus bounded by the value of K, the carrying capacity. In the model described by Omonyi (2014) the parameter r, which represents the difference between the per-capita birth and death rate, plays a crucial role. It is important to note that, in practice, the values of r and K are influenced by various socio-economic and demographic factors in any given population.

2.1.1 Parameter Sensitivity Analysis

To study the sensitivity of the parameters in the population growth model, the expression for the population growth (equation 2.1) is differentiated with respect to each of the parameters. Sensitivity analysis helps to understand how small changes in these parameters impact the overall population dynamics. The local sensitivity function is defined as:

$$\frac{\partial N(t,p)}{\partial p_i} = \left(\frac{\partial f(t,p)}{\partial p_i}\right) + \left(\frac{N(t,p)}{\partial t}\right) \tag{2.3}$$

where p_i represents the parameters of interest (e.g., r and K), and $f(t,p)$ represents the right-hand side of the logistic growth equation.

The sensitivity matrix is denoted as:

$$S(t,p) = \left(\frac{\partial N(t,p)}{\partial p}\right) \in \mathbb{R}^{m\times n} \tag{2.4}$$

where m represents the time instants (years) for which the population is enumerated, and n is the number of parameters of interest. Local sensitivity analysis involves calculating the partial derivatives of the population growth model with respect to the parameters.

The time-dependent sensitivity functions are computed for each parameter using the following expressions:

$$\begin{aligned} \frac{\partial N(t)}{\partial r} &= N(t)(1-2\frac{N(t)}{K}) \\ \frac{\partial N(t)}{\partial K} &= rN(t)\left(1-2\frac{N(t)}{K}\right) - \frac{rN^2(t)}{K^2} \end{aligned} \tag{2.5}$$

Since the initial values for the population do not depend on the parameters, the initial conditions satisfy $S(0,p) = 0$. These equations are then solved to obtain the sensitivity functions for r and K (see Banks et al., 2009; National, 2009; Tavener et al., 2011; Giaimo et al., 2022).

2.1.2 Evaluation of Parameter Sensitivities

The sensitivity of a parameter provides insight into how much a small change in that parameter affects the population. In this study, the sensitivity of the population with respect to the parameters r and K is evaluated using the following integral metric:

$$\Phi = \int_{t_0}^{t_f} |S(t,p)| dt \tag{2.6}$$

where t_0 and t_f are the start and end time instants. This sensitivity metric helps rank the influence of the parameters on the population growth. In this work, the metric is used to evaluate the impact of changes in r and K on the population dynamics. All calculations were performed using MATLAB R2023a installed on a personal computer with Intel(R) Core(TM) i5-2600 CPU @ 2.30 GHz and 8.00 GB RAM, running on Windows 8.1.

3. Results and Discussion

In the simulation, nominal parameter values were used: $r \approx 0.093009$, $K \approx 1.694 \times 10^{10}$, $A \approx 10{,}979.62$ and $k \approx 0.911184881$. These values are based on estimates from prior studies. The modeling and analysis in this work utilized population data from the National Population Commission, as detailed in Table 1.

Table 1. Actual values of population as specified in the gazette

Year	Time	Actual	Logistic	Sensitivity
2007	0	1,543,293	1,543,293	143527.0452
2008	1	1,693,706	1,693,706	157514.1336
2009	2	1,858,777	1,858,777	172864.0015
2010	3	2,039,937	2,039,934	189709.3651
2011	4	2,238,752	2,238,744	208195.8528
2012	5	2,456,945	2,456,928	228483.2589
2013	6	2,696,403	2,696,372	250746.9188
2014	7	2,959,199	2,959,147	275179.2161
2015	8	3,247,608	3,247,526	301991.2366
2016	9	3,564,126	3,564,002	331414.5804

Simulations were conducted using this data, and the results are presented in Figures 1, 2 & 3.

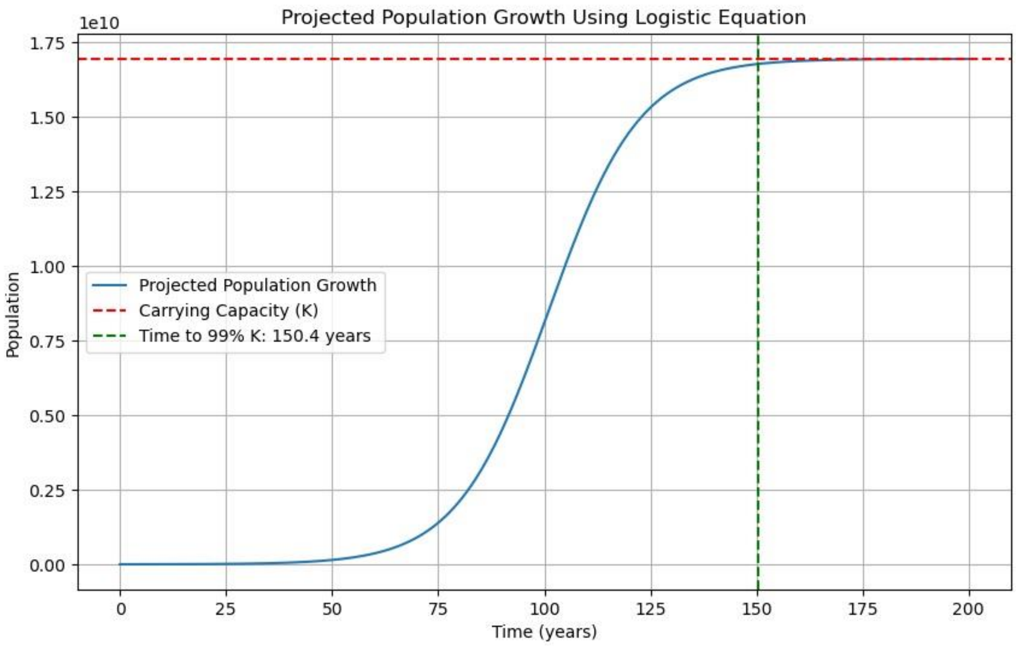

Figure 1. A plot showing projected growth using logistic equation

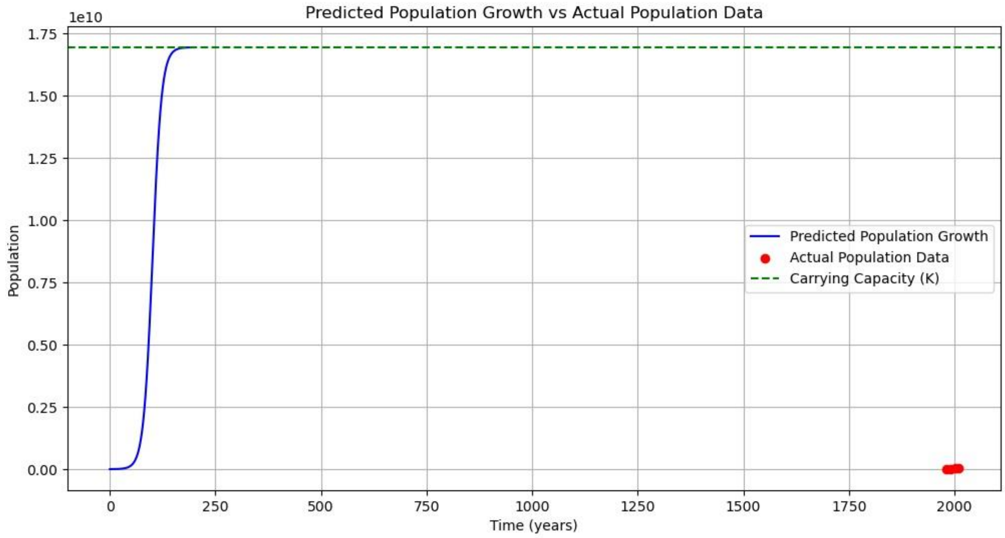

Figure 2. A plot showing predicted growth vs actual population data

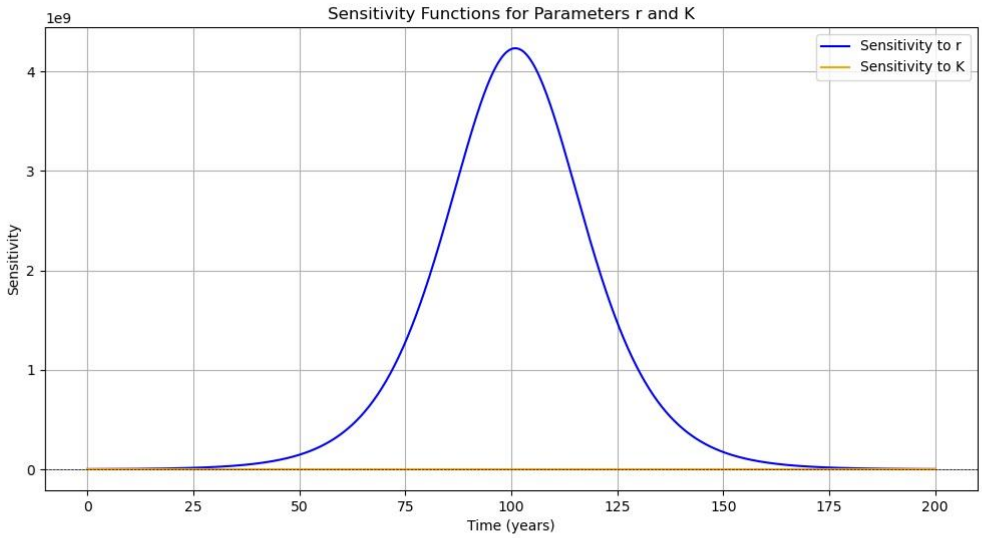

Figure 3. A plot showing sensitivity functions for parameters of Φ_K and Φ_r

The sensitivity of the population to changes in the parameters r and K was assessed. The sum of the sensitivity values for the population per-capita birth and death rates were calculated, yielding:

$$\Phi_r \approx 1.8209 \times 10^{11} \text{ and } \Phi_K \approx 0.999817$$

This indicates that the population is highly sensitive to variations in the birth rate parameter r, while the carrying capacity K has little influence under the given conditions. The point of inflection, representing the time when population growth begins to slow down, was computed to occur at $t = 99$ years (i.e., in the year 2107, starting from the year 2007 as the baseline). This suggests that the population will continue to grow rapidly for the next several decades before stabilizing as it approaches the carrying capacity.

3.1 Projected Population Growth

Figure 1 shows the projected population growth over time, where the blue line represents the population growth curve. The green dotted line highlights the period during which the population approaches the carrying capacity. This projection assumes that the population growth remains unchecked and continues along its current trajectory. The results suggest that, within the next ~120 years from the baseline (2007), the population is likely to reach levels that impose a significant strain on the city's resources, particularly in terms of food, water, healthcare, and infrastructure.

3.2 Sensitivity Analysis

Figure 3 illustrates the sensitivity of the population to changes in the parameters r and K. The results show that the parameter r (the difference between the birth and death rates) has a far greater influence on population growth than K (the carrying capacity). This implies that even small changes in the per-capita birth rate could lead to significant fluctuations in the overall population size, especially in the short term. Given the current trajectory, Abuja is expected to face challenges in managing its population growth, particularly as the demand for social services such as education, healthcare, and sanitation continues to rise. The rapid population growth, if unchecked, could lead to a depletion of resources within the next 500 years, under the assumption that no major catastrophes or extreme events (e.g., epidemics, environmental disasters) alter the death rate significantly.

3.3 Implication to Policy and Planning

These findings emphasize the importance of population control measures, particularly those that address the birth rate. Efforts to reduce the birth rate, such as family planning and public health education, could have a significant impact on slowing the population growth and mitigating its strain on resources. In particular, improving access to birth control and healthcare services in rural areas, where poverty and illiteracy are common, could help reduce the birth rate and contribute to more sustainable growth. The results also suggest that periodic re-evaluation of the model parameters is necessary to ensure accurate predictions. Factors such as immigration, emigration, and undocumented individuals could introduce biases into the model, making frequent updates to the population data essential for improving the precision of future predictions.

4. Conclusion

The results of this study demonstrate the significant impact of parameter variations on population dynamics, particularly the sensitivity of the population to changes in the birth rate. The analysis shows that, without intervention, the rapid population growth in Abuja could lead to unsustainable resource use within the next few centuries. The Logistic growth model, though simple, proved to be an effective tool for predicting population trends and identifying key parameters that influence population changes over time. Efforts to manage population growth through family planning, healthcare, and education must be intensified to ensure that Abuja can adequately plan for its future. Continued collection and analysis of demographic data, along with the development of more sophisticated models, will help improve the accuracy of predictions and aid in policy planning. The sensitivity analysis highlights the need for particular attention to the birth rate as a key driver of population growth. A reduction in the per-capita birth rate, while not a panacea, would alleviate some of the pressures on resources and make it easier to meet the social and economic needs of the population. This study also calls for further research into models that can account for additional factors, such as migration and urbanization, which also influence population dynamics.

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