Bayesian Generalized Least Squares with Autocorrelated Error

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Abstract

Autocorrelation plays a significant role in both time series and cross-sectional data. More often than none it rendered the inference of parameter estimates invalid and those other statistics that use the parameters. This study investigates the asymptotic behaviour of generalised least squares with Autocorrelated errors cum Markov Chain Monte Carlo simulation. Bias and Mean Squares Error criteria were used to evaluate the finite properties of the estimator. The following sample sizes: 25, 50, 100, and 250 were constructed and used. Thus 11,000 simulations with varying levels of Autocorrelated error were carried out. This is subjected to the level of convergence. Bias and Mean Squares Error criteria revealed improving performance asymptotically regardless of the level of Autocorrelated error. The study observed that asymptotic consistency and efficiency were obtained at the large sample which obeys the law of large number and points to the fact that variance of error terms tend towards zero and distribution tends to be normal when the law of large number is applied. In line with the above assertion, the study had developed novel Bayesian Generalized Least Squares Autocorrelated Estimator (BGLSAE) that capture the presence of autocorrelation in the dataset. The study, therefore recommended that large samples should be obtained to make the inferences stable.

Keywords: Autocorrelation, Autocorrelation, Bayesian Inference, Gibbs Sampler and Markov Chain Monte Carlo Method.

1. Introduction

In a Bayesian framework, the knowledge about the parameters of the model is described by a probability distribution (Oloyede et al 2013). The generalized least squares (GLS) estimation does not bring about the uncertainty of model estimates for both error variance and variance of parameters $\hat{\beta}$ (Reis et al 2005). Hisashi (2002), Chib and Greenbaerg (1994), and Chib (1993) examined Autocorrelated error of regression model in time series data structure via Bayesian framework. First-order Autocorrelated error term with assumed stationary in Bayesian experimental framework was examined in Hisashi (2002), where he compared maximum likelihood estimation with Bayesian estimation and concluded that Bayesian estimator has robust performance over maximum likelihood estimator. This was explored in view of time series data structure with time inclusive. Skiera, Reiner, and Albers (2018) described autocorrelation in regression analysis as residuals that correlate with each other. They further pointed out that autocorrelation leads to a situation where the predicted values are too high for some periods and too low for others and thus, a series of negative residuals alternate with a series of positive residuals.

Akpan and Moffat (2018) opined that if the assumption of no correlation in the error term is violated, then, the underlying model would be rendered invalid with the standard errors of the parameters becoming biased. Moreover, if the errors are correlated, the least squares estimators are inefficient and the estimated variances were not appropriate. They thereby evidently proved that generalized least squares is a panacea for the weaknesses of ordinary least squares and accounted for the presence of autocorrelation in the error terms.

Mukherjee and Laha (2019) submitted that when the disturbance term exhibits serial correlation, the value of the standard error of the parameter estimates are affected and the predictions based on ordinary least square estimates (OLS) will be inefficient, in the econometrics technique. Although the parameter estimates of ordinary least squares (OLS) of are statistically unbiased in the sense that their expected value is equal to the true parameter. Shalabh (2020) conducted a study on regression analysis and he submitted that the carryover effect, at least in part, is an important source of autocorrelation. He further explained that autocorrelation could be introduced based on the effect of deletion of some variables, misspecification of the form of relationship can also introduce autocorrelation in the data and the presence of measurement errors on the dependent variable may also introduce the autocorrelation in the data.

Lusomba (2020) observed that residuals from the estimation impulse response functions directly using linear regressions which are called Local Projections (LP) residuals are autocorrelated. Lazarus et al. (2018) claimed that LP has to be estimated with heteroscedasticity and autocorrelation consistent (HAC)/standard errors because Generalized least Squares (GLS) estimates would be inconsistent. He showed that under standard time series assumptions, the autocorrelation process is known and autocorrelation can be corrected for using GLS. Moreover, consistency and asymptotically normality of the LP GLS estimator, as well as the asymptotic efficiency of LP GLS relative to LP OLS.

Miranda-Agrippino and Ricco (2018) submitted that since the autocorrelation process is known, LP GLS can be estimated using fully Bayesian methods and Bayesian LP have many advantages such as allowing the researcher to incorporate prior information for impulse responses at each horizon.

The joint posterior distribution is the product of likelihood and prior which is divided by normalizing constant, thus normalizing constant often portend computationally intensive. It is usually assumed equal to unity. Meanwhile, the Markov Chain Monte Carlo simulation technique that draws correlated samples of parameters from the joint posterior distribution with normalizing constant set to unity would proffer a solution to the problem of intensive computation, Gilks et al (1996).

To overcome those problems, the study adopted a fully Bayesian approach, which automatically averages over our uncertainty in the model parameters. In this paper, variance-covariance Autocorrelated error structure was incorporated into Bayesian generalized least squares. We extend our study to the multidimensional and more complicated cases and carry out simulation using Markov Chain Monte Carlo (MCMC); this paper also examined the finite sample properties of the estimator. These are the gaps that this study decides to fill.

2. Set-up and Model Designs

Let $y = X\beta + u$ with $u \sim N(0, \frac{1}{1-\rho^2}\Sigma)$ where Σ is a positive definite matrix of order $n \times n$ where $\frac{1}{1-\rho^2}\Sigma$ is covariance matrix of Autocorrelated errors.

$$y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + u_t \quad (1)$$

$$u_t = u_t \rho + v_t \quad (2)$$

The variance-covariance matrix of Autocorrelated error as in Gary *etal* (2007) is shown below:

$$E(UU') = \frac{1}{1-\rho^2}\begin{bmatrix} 1 & \rho & \rho^2 & . & . & . & \rho^{n-1} \\ \rho & 1 & \rho & . & . & . & \rho^{n-2} \\ \rho^2 & \rho & 1 & . & . & . & \rho^{n-3} \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ \rho^{n-1} & \rho^{n-2} & \rho^{n-3} & . & . & . & \end{bmatrix} \quad (3)$$

$$E(UU') = \frac{1}{1-\rho^2}\Sigma \quad (4)$$

The likelihood function of θ, where $\theta = (\beta, \rho, \sigma^2)$ given the sample vector $X_1, X_2 = (1,2,\ldots,n)'$ and $y = (y_1, y_2, \ldots, y_n)'$ is expressed as

$$L(y|\theta,\rho,\sigma,X) = \left(\frac{2\pi\sigma^2}{1-\rho^2}\right)^{-\frac{n}{2}} \prod_{i=1}^{n} \exp\left\{-\frac{1}{\frac{2\sigma^2}{1-\rho^2}} \sum_{i=1}^{n} [y_i - x\beta]^2\right\} \quad (5)$$

$$L(y|\theta,\rho,\sigma,X) = (2\pi\sigma^2)^{-n/2}(1-\rho^2)^{1/2} \prod_{i=1}^{n} \exp\left\{-\frac{(1-\rho^2)^{-1}}{2\sigma^2} \sum_{i=1}^{n} [y_i - x\beta]^2\right\} \quad (6)$$

Incorporating Autocorrelated error covariance matrix (Σ) in equation (4) into the likelihood function of equation (6) resulted in to

$$L(y|\theta,\rho,\sigma,X) = (2\pi\sigma^2)^{-\frac{n}{2}} \prod_{i=1}^{n} \left|(1-\rho^2)^{\frac{1}{2}}\Sigma^{-1}\right| \exp\left\{-\frac{(1-\rho^2)^{-1}}{2\sigma^2} \sum_{i=1}^{n} (y_i - x\beta)'\Sigma^{-1}(y_i - x\beta)\right\} \quad (7)$$

To derive the full Bayesian density, the study conjugated the error density function, Equation (6) with multivariate normal distribution, and inverse-gamma distribution. Marginal posterior density was obtained by marginalizing the joint posterior density with respect to each parameter of interest. The study adopted prior density $\pi(\beta_0, \beta_1, \beta_2, \rho, \sigma) = \pi(\beta_0)\pi(\beta_1)\pi(\beta_2)\pi(\rho)\pi(\sigma)$.-Thus normal distribution is considered for β_s, while inverse gamma is considered for σ and a uniform distribution is considered for ρ such that

$$\pi(\beta) \propto (2\pi\sigma^2)^{-\frac{n}{2}}(1-\rho^2)^{\frac{1}{2}} \exp\left\{-\frac{1}{2\sigma^2}(\beta-\mu)^2\right\}, \beta > 0; \quad (8)$$

$$\pi(\sigma^2) \propto (\sigma^2)^{-a_1+1} \exp(-b_1/\sigma^2), \sigma^2 > 0 \quad (9)$$

$$\pi(\rho) \propto c \quad (10)$$

c is constant $-1 < \rho < 1$

The posterior distribution of $\theta = (\beta_0, \beta_1, \beta_2, \rho, \sigma)$, considering independence among the parameters is given by:

$$\pi(\beta_0, \beta_1, \beta_2, \rho, \sigma | X, y) \propto$$

$$(2\pi\sigma^2)^{-\frac{n}{2}}(\sigma^2)^{-(a_1-1-n/2)} \exp\left\{-\frac{1}{2\sigma^2}\ (\beta-\mu)^2\right\}\prod_{i=1}^{n}\left|(1-\rho^2)^{1/2}\Sigma\right| \exp\{-\frac{(1-\rho^2)^{-1}}{\sigma^4}(b_1 + \frac{1}{2}\sum_{i=1}^{n}(y_i - x\beta)'\Sigma^{-1}(y_i - x\beta)\} \quad (11)$$

where a_1, b_1 are the hyper-parameters for the inverse-gamma which were excluded for β-parameters since they would be estimated from the data and may be arbitrarily small leading to problems that may eventually affect the inferences. Integrating the posterior $\pi(\beta, \rho, \sigma | X, y)$ with respect to σ^2, thus we have joint a posterior distribution for (β, ρ)

$$\pi(\beta_0, \beta_1, \beta_2, \rho, \sigma | X, y) \propto$$

$$(2\pi)^{-\frac{n}{2}} \exp\left\{-\frac{1}{2}\ (\beta-\mu)^2\right\}\prod_{i=1}^{n}\left|(1-\rho^2)^{1/2}\Sigma\right| \exp\{-b_1 - \frac{(1-\rho^2)^{-1}}{2}\sum_{i=1}^{n}(y_i - x\beta)'\Sigma^{-1}(y_i - x\beta)\}^{-(a_{1-n/2})} \quad (12)$$

This yields the following full conditional density of the parameters, ρ and σ:

$$\pi(\beta_0|\sigma^2, \rho, \theta, X, y) \propto \exp\left\{-\frac{1}{2}\ (\beta_0-\mu)^2\right\}\prod_{i=1}^{n}\left|(1-\rho^2)^{1/2}\Sigma\right| \exp\{-\frac{(1-\rho^2)^{-1}}{2}\sum_{i=1}^{n}(y_i - x\beta)'\Sigma^{-1}(y_i - x\beta)\}^{-(a_{1-n/2})} \quad (13)$$

$$\pi(\beta_1|\sigma^2, \rho, \theta, X, y) \propto \exp\left\{-\frac{1}{2}\ (\beta_1-\mu)^2\right\}\prod_{i=1}^{n}\left|(1-\rho^2)^{1/2}\Sigma\right| \exp\{-\frac{(1-\rho^2)^{-1}}{2}\sum_{i=1}^{n}(y_i - x\beta)'\Sigma^{-1}(y_i - x\beta)\}^{-(a_{1-n/2})} \quad (14)$$

$$\pi(\beta_2|\sigma^2, \rho, \theta, X, y) \propto \exp\left\{-\frac{1}{2}\ (\beta_2-\mu)^2\right\}\prod_{i=1}^{n}\left|(1-\rho^2)^{1/2}\Sigma\right| \exp\{-\frac{(1-\rho^2)^{-1}}{2}\sum_{i=1}^{n}(y_i - x\beta)'\Sigma^{-1}(y_i - x\beta)\}^{-(a_{1-n/2})} \quad (15)$$

$$\pi(\rho|\beta, \sigma^2, X, y) \propto cnst$$

. $$((\sigma^2|\beta, \rho, X, y) \propto \prod_{i=1}^{n}\left|(1-\rho^2)^{\frac{1}{2}}\Sigma\right|(b_1 + \frac{(1-\rho^2)^{-1}}{2}\sum_{i=1}^{n}(y_i - x\beta)'\Sigma^{-1}(y_i - x\beta)\}^{-\left(a_{2+\frac{n}{2}}\right)} \quad (16)$$

Gibbs sampling Algorithm update was performed on the full conditional distribution of

$$\beta \propto MVN(\beta, \sigma^2(1-\rho^2)^{-1}(X'\Sigma^{-1}X)^{-1}) \quad (17)$$

$$\sigma^2 \propto$$

$$IG\left(a_1 + \frac{n}{2}, b_1 + \frac{(1-\rho^2)^{-1}}{2}\sum_{i=1}^{n}(y_i - x\beta)'\Sigma^{-1}(y_i - x\beta)\right). \quad (18)$$

$$\rho \propto \text{unif}(-1, 1) \quad (19)$$

2.1 Data Generation Processes

The study adopted Gibbs sampler Experiments. The sample sizes were specified with 4 sets as follows: 25, 50, 100 and 250. ρ was systematically selected as - 0.9, -0.6, -0.3, -0.1, 0.1, 0.3, 0.6, 0.9. The covariates X_1 and X_2 are generated using a uniform distribution. The error term U was generated based on $E(UU') \neq 0$. Thereafter, the study incorporated it into the model and the parameters β_0, β_1 and β_2 were set at 10, 1 and 1 respectively to generate variable y. The number of replications of the experiment was set at 11,000 with a burn-in of 1000 which specified the draws that were discarded to remove the effect of the initial values. The thinning was set at 5 to ensure the removal of the effect of autocorrelation in the MCMC simulation.

3. Results and Discussion

This study presented serially correlated error truncated linear model, the parameter estimates were obtained through the posterior point estimate of Gibbs sampler Algorithm using Marcov Chain Monte Carlo simulation.

Setting ρ equal zero indicates no Autocorrelated errors, whereas $\rho's$ were set between 0.1 to 0.9 to capture various effect of Autocorrelated error in the regression inferences. This was observed asymptotically with the use of measurement metrics (Bias and Mean Squares error). Thus from the outcome of the study, the estimator performed better as well when $\rho = 0$.

Table 1 revealed the outcome of the estimation of Bayesian GLS Autocorrelated Error (GLSAE) linear model. It showed that the bias for β_0 and $\hat{\beta}_1$ decreased algebraically as the ρ decreased from 0.9 to 0.1 across all the sample sizes from 25 to 250, though this is mainly for positive ρ. For the negative ρ, the bias for $\hat{\beta}_0$ and $\hat{\beta}_1$ increased algebraically as the ρ decreases from -0.1 to -0.9. We observed consistency for parameters $\hat{\beta}_0$ and $\hat{\beta}_1$ across all the sample sizes. Meanwhile, the bias for $\hat{\beta}_2$ showed inconsistency at sample sizes 25 and 50, but in sample size 100, the bias reduced as the ρ reduced at positive and negative ρ, thus the study observed consistency.

Table 1. Performances of the Bayesian Generalized Least Squares Autocorrelated Errors (GLSAE) on the basis of Bias criterion

Samples	ρ	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$
25	0.9	0.0133	-0.0025	-0.0001
	0.6	0.0116	-0.0021	0.0001
	0.3	0.0106	-0.0018	0.0004
	0.1	0.0104	-0.0018	0.0006
	0	**0.0105**	**-0.0018**	0.0007
	-0.1	0.0106	-0.0018	0.0007
	-0.3	0.0112	-0.0018	0.0009
	-0.6	0.0128	-0.002	0.0011
	-0.9	0.0148	-0.0023	0.0012
50	0.9	0.0120	-0.0030	0.0004
	0.6	0.0105	-0.0025	0.0006
	0.3	0.0093	-0.0022	0.0008
	0.1	0.0088	-0.0021	0.0009
	0	**0.0086**	**-0.0020**	**0.0009**
	-0.1	0.0085	-0.0020	0.0011
	-0.3	0.0086	-0.0021	0.0013
	-0.6	0.0092	-0.0023	0.0016
	-0.9	0.0104	-0.0026	0.0019
100	0.9	0.0204	-0.0028	-0.0009
	0.6	0.0178	-0.0024	-0.0007
	0.3	0.0158	-0.0019	-0.0005
	0.1	0.0151	-0.0018	-0.0003
	0	**0.0149**	**-0.0017**	**-0.0003**
	-0.1	0.0149	-0.0017	-0.0003
	-0.3	0.0152	-0.0017	-0.0002
	-0.6	0.0167	-0.0017	-0.0001
	-0.9	0.0191	-0.0019	-0.0001
250	0.9	0.0151	-0.0024	-0.0004
	0.6	0.0132	-0.0020	-0.0003
	0.3	0.0121	-0.0018	-0.0002
	0.1	0.0119	-0.0017	-0.0002
	0	**0.0121**	**-0.0017**	**-0.0002**
	-0.1	0.0123	-0.0017	-0.0002
	-0.3	0.0131	-0.0017	-0.0002
	-0.6	0.0149	-0.0019	-0.0002
	-0.9	0.0173	-0.0021	-0.0002

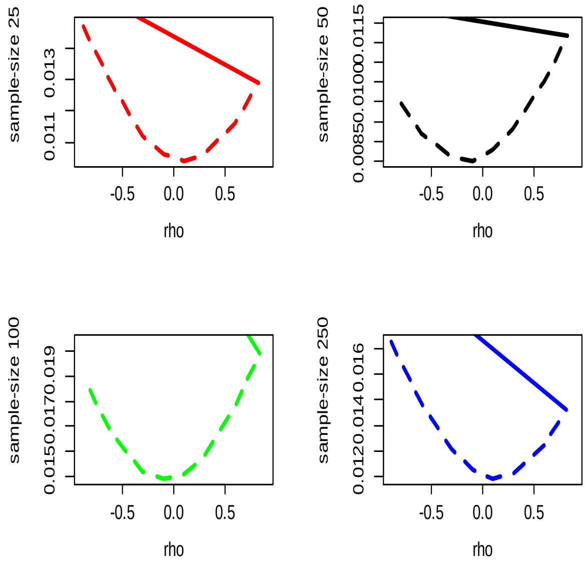

Figure 1: Performances of the Bayesian Generalized Least Squares Autocorrelated Errors (GLSAE) on the basis of Bias criterion

Table 2 and Figure 2 revealed the mean squared error (MSE) criterion, the mean squares error for $\hat{\beta}_0$, $\hat{\beta}_1$ and $\hat{\beta}_2$ decreased algebraically as the ρ decreased from 0.9 to 0.1 across all the sample sizes, but the mean squares error for $\hat{\beta}_0$, $\hat{\beta}_1$ and $\hat{\beta}_2$ increased algebraically as the ρ decreased from -0.1 to -0.9 across all the sample sizes considered in the study. The study confidently concluded that there is efficiency with the decrease in ρ leading to a decrease in mean squares error.

Table 2. Performances of the Bayesian GLSAE on the basis of MSE criterion

Samples	ρ	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$
25	0.9	1.5562	0.0128	0.0264
	0.6	1.1780	0.0101	0.0201
	0.3	0.9578	0.0088	0.0161
	0.1	0.8988	0.0088	0.0146
	0	**0.8956**	**0.0090**	**0.0143**
	-0.1	0.9098	0.0094	0.0142
	-0.3	0.9912	0.0106	0.0148
	-0.6	1.2448	0.0136	0.0175
	-0.9	1.6563	0.0181	0.0225
50	0.9	0.7424	0.0070	0.0119
	0.6	0.5590	0.0054	0.0088
	0.3	0.4213	0.0045	0.0066
	0.1	0.3639	0.0043	0.0056
	0	**0.3446**	**0.0043**	**0.0053**
	-0.1	0.3314	0.0044	0.0050
	-0.3	0.3238	0.0049	0.0049
	-0.6	0.3589	0.0061	0.0054
	-0.9	0.4501	0.0082	0.0068

100	0.9	0.3801	0.0037	0.0046
	0.6	0.2862	0.0028	0.0035
	0.3	0.2265	0.0022	0.0028
	0.1	0.2058	0.0019	0.0026
	0	**0.2011**	**0.0018**	**0.0025**
	-0.1	0.2002	0.0018	0.0026
	-0.3	0.2098	0.0019	0.0027
	-0.6	0.2528	0.0022	0.0034
	-0.9	0.3299	0.0029	0.0044
250	0.9	0.1306	0.0013	0.0019
	0.6	0.0998	0.0009	0.0014
	0.3	0.0836	0.0008	0.0013
	0.1	0.0809	0.0007	0.0011
	0	**0.0821**	**0.0007**	**0.0011**
	-0.1	0.0849	0.0007	0.0011
	-0.3	0.0954	0.0008	0.0012
	-0.6	0.1234	0.0009	0.0015
	-0.9	0.1662	0.0013	0.0019

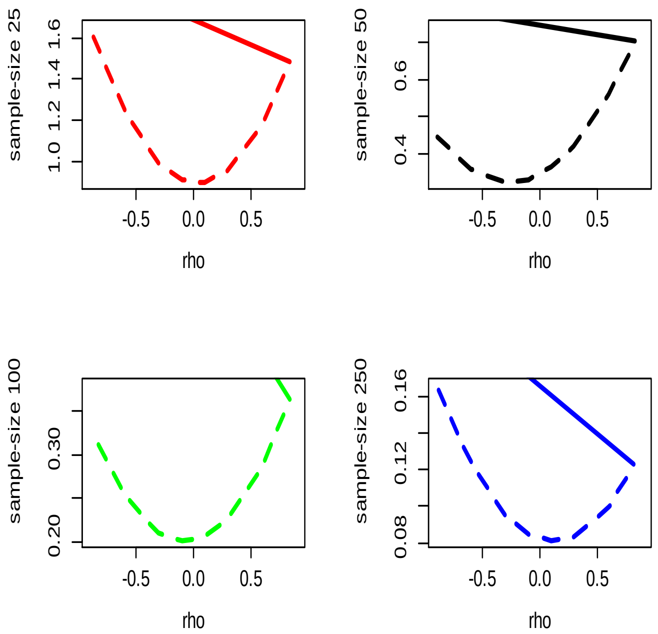

Figure 2: Performances of the Bayesian GLSAE on the basis of MSE criterion

Table 3 and Figure 3 revealed the asymptotic performances of the Bayesian Generalized Least Squares Autocorrelated Error on the basis of bias, at $\hat{\beta}_0$, the bias reduced algebraically as the sample sizes increased from 25 to 50 thereafter increased as the sample sizes increased from 50 to 100. After sample size 100, the study observed consistency as the sample sizes increased from 100 up to 250, the bias recorded were reduced across all the ρ's deemed in the study. For $\hat{\beta}_1$ and $\hat{\beta}_2$, the bias increased algebraically as the sample sizes increased from 25 to 50 thereafter decreased as the sample size increased from 50 to 250. Therefore, the study observed consistency as the sample sizes increased from 50 up to 250, the bias recorded were reduced across all the ρ's.

Table 3. Asymptotic Performances of the Bayesian GLSAE on the basis of Bias

Samples	ρ	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$
25	0.9	0.0133	-0.0025	-0.0001
50	0.9	0.012	-0.003	0.0004
100	0.9	0.0204	-0.0028	-0.0009
250	0.9	0.0151	-0.0024	-0.0004
25	0.6	0.0116	-0.0021	0.0001
50	0.6	0.0105	-0.0025	0.0006
100	0.6	0.0178	-0.0024	-0.0007
250	0.6	0.0132	-0.002	-0.0003
25	0.3	0.0106	-0.0018	0.0004
50	0.3	0.0093	-0.0022	0.0008
100	0.3	0.0158	-0.002	-0.0005
250	0.3	0.0121	-0.0018	-0.0002
25	0.1	0.0104	-0.0018	0.0006
50	0.1	0.0088	-0.0021	0.0009
100	0.1	0.0151	-0.0018	-0.0003
250	0.1	0.012	-0.0017	-0.0002
25	0	**0.0105**	**-0.0018**	**0.0007**
50	0	**0.0086**	**-0.0020**	**0.0009**
100	0	**0.0149**	**-0.0017**	**-0.0003**
250	0	**0.0121**	**-0.0017**	**-0.0002**
25	-0.1	0.0106	-0.0018	0.0007
50	-0.1	0.0085	-0.002	0.0011
100	-0.1	0.0149	-0.0017	-0.0003
250	-0.1	0.0123	-0.0017	-0.0002
25	-0.3	0.0112	-0.0018	0.0009
50	-0.3	0.0086	-0.0021	0.0013
100	-0.3	0.0152	-0.0017	-0.0002
250	-0.3	0.0131	-0.0017	-0.0002
25	-0.6	0.0128	-0.002	0.0011
50	-0.6	0.0092	-0.0023	0.0016
100	-0.6	0.0167	-0.0017	-0.0001
250	-0.6	0.0149	-0.0019	-0.0002
25	-0.9	0.0148	-0.0023	0.0012
50	-0.9	0.0104	-0.0026	0.0019
100	-0.9	0.0191	-0.002	-0.0001
250	-0.9	0.0173	-0.0021	-0.0002

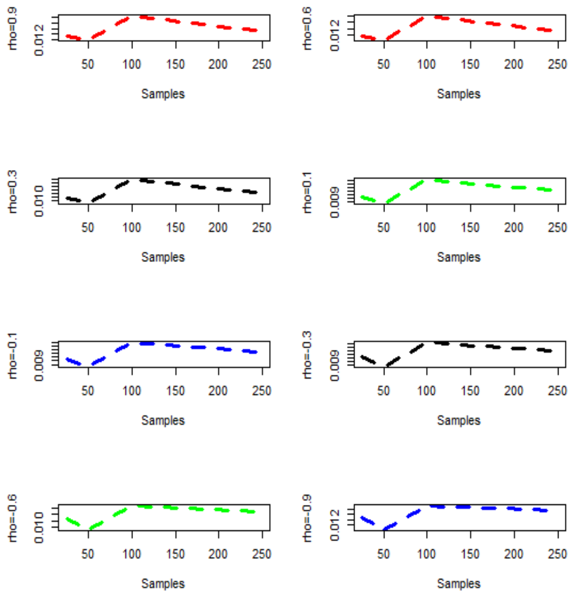

Figure 3: Asymptotic Performances of the Bayesian GLSAE on the basis of Bias

Table 4 and Figure 4 revealed the mean squared error criterion, the mean squares error for $\hat{\beta}_0$, $\hat{\beta}_1$ and $\hat{\beta}_2$ decreased algebraically as the sample sizes increased from 25 up to 250 across all the ρ's deemed in the study. The study found out that there is efficiency with the decrease in mean squares error asymptotically.

Table 4. Asymptotic Performances of the Bayesian GLSAE on the basis of MSE criterion

Samples	ρ	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$
25	0.9	1.5562	0.0128	0.0264
50	0.9	0.7424	0.007	0.0119
100	0.9	0.3801	0.0037	0.0046
250	0.9	0.1306	0.0013	0.0019
25	0.6	1.178	0.0101	0.0201
50	0.6	0.5539	0.0054	0.0088
100	0.6	0.2862	0.0028	0.0035
250	0.6	0.0998	0.001	0.0014
25	0.3	0.9578	0.0088	0.0161
50	0.3	0.4213	0.0045	0.0066
100	0.3	0.2265	0.0022	0.0028
250	0.3	0.0836	0.0008	0.0012
25	0.1	0.8988	0.0088	0.0146
50	0.1	0.3639	0.0043	0.0056
100	0.1	0.2058	0.0019	0.0026
250	0.1	0.081	0.0007	0.0011
25	0	**0.8956**	**0.0090**	**0.0143**
50	0	**0.3446**	**0.0043**	**0.0053**
100	0	**0.2011**	**0.0018**	**0.0025**
250	0	**0.0821**	**0.0007**	**0.0011**

25	-0.1	0.9099	0.0094	0.0142
50	-0.1	0.3314	0.0044	0.005
100	-0.1	0.2002	0.0018	0.0026
250	-0.1	0.0849	0.0007	0.0011
25	-0.3	0.9912	0.0106	0.0148
50	-0.3	0.3238	0.0049	0.0049
100	-0.3	0.2098	0.0019	0.0027
250	-0.3	0.0954	0.0008	0.0012
25	-0.6	1.2448	0.0136	0.0175
50	-0.6	0.359	0.0062	0.0054
100	-0.6	0.2528	0.0022	0.0034
250	-0.6	0.1234	0.001	0.0015
25	-0.9	1.6563	0.0181	0.0225
50	-0.9	0.4501	0.0082	0.0068
100	-0.9	0.33	0.0029	0.0044
250	-0.9	0.1662	0.0013	0.002

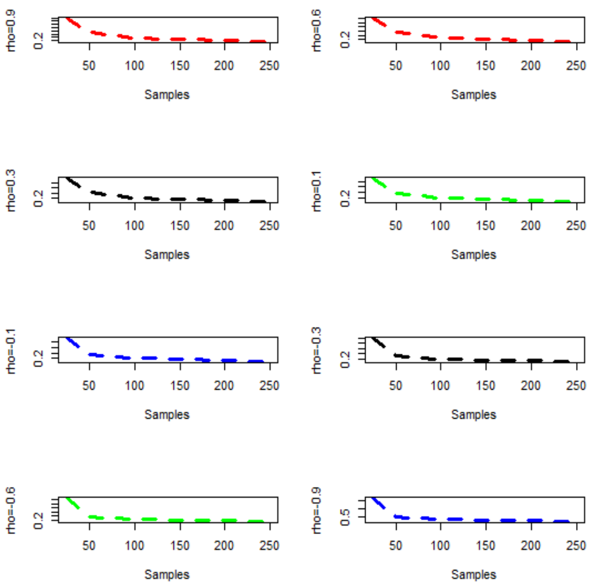

Figure 4: Asymptotic Performances of the Bayesian GLSAE on the basis of MSE criterion

4. Conclusion

The study observed that modelling Autocorrelated Error in a full Bayesian improves the precision of the inferences of the estimates. The study found it difficult to compare the findings with literature, this is because their works were based on time series data while the study was on cross sectional data. The study observed that asymptotic consistency and efficiency were obtained at large sample which obey the law of large number and point to the fact that variance of error terms tend towards zero and distribution tends to normal when law of large number is applied. In line with the above assertion, the study had developed novel Bayesian Generalized Least Squares Autocorrelated Estimator (BGLSAE) that capture the presence of autocorrelation in the dataset. More importantly, the study varied the

Autocorrelated errors and examined the work asymptotically. The study concluded that asymptotically there exists consistency and efficiency in the estimation. The approach can be applied to further studies in the area of other econometrics, biometrics and time series models. The study therefore recommended that large samples should be obtain to make the inferences stable.

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